Matrix multiplication is different from regular number multiplication.

For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ calculate:

(a) $AD = \begin{bmatrix} 2 & -2 \\ 6 & -4 \end{bmatrix} = \left[\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} \right] = \left[2\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 4 \end{bmatrix}, 0\begin{bmatrix} 1 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 2 \\ 4 \end{bmatrix} \right]$

Double first column, negate second column

(b) $DA = \begin{bmatrix} 2 & 4 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2(1)+0(3) & 2(2)+0(4) \\ 0(1)-1(3) & 0(2)-1(4) \end{bmatrix}$

Double first <u>row</u>, negate second <u>row</u>

(c) Find a matrix C so that ~~$AC = CA$~~. $DC = CD$

$C = \begin{bmatrix} x_1 & 0 \\ 0 & x_4 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} 2x_1 & 2x_3 \\ -x_2 & -x_4 \end{bmatrix}$

$\begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2x_1 & -x_3 \\ 2x_2 & -x_4 \end{bmatrix}$

so
$2x_1 = 2x_1 \to x_1$ is free
$-x_2 = 2x_2 \to x_2 = 0$
$2x_3 = -x_3 \to x_3 = 0$
$-x_4 = -x_4 \to x_4$ is free

(d) Find a matrix I so that $AI = IA = A$.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} x_1+2x_2 & x_3+2x_4 \\ 3x_1+4x_2 & 3x_3+4x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\begin{matrix} x_1 + 2x_2 = 1 \\ 3x_1 + 4x_2 = 3 \end{matrix} \Rightarrow \begin{matrix} x_1 + 2x_2 = 1 \\ 0 \quad -2x_2 = 0 \end{matrix} \Rightarrow \begin{matrix} x_1 = 1 \\ x_2 = 0 \end{matrix} \quad \dots \quad \begin{matrix} x_3 = 0 \\ x_4 = 1 \end{matrix}$

(e) Find a matrix E so that $DE = ED = I$

$E = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} 2x_1 & 2x_3 \\ -x_2 & -x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2x_1 = 1 \qquad 2x_3 = 0$

$-x_2 = 0 \qquad -x_4 = 1$

(f) List 10 ways to know if a matrix B could have an "E" such that $BE = EB = I$.

Rows are indep. RREF(B) has pivot in every row

Cols are indep. RREF(B) has pivot in every col

... see Ch 2.3

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2.1.1(HW2.1#11) Find a matrix B so that $DB = BD$ where $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$B = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

Don't use $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Can you find <u>all</u> of them?

2.1.2 (HW2.1#10) Find a matrix X so that $AX = 0$ where $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

$X = \begin{bmatrix} & \\ & \end{bmatrix}$

Don't use $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Can you find <u>all</u> of them?

2.2.1 Can there be a matrix E such that $EA = I$? (If so, find E; if not explain why not.)

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find E or show it doesn't exist

$A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

2.2.2 Can there be a matrix E such that $ED = I$? (If so, find E; if not explain why not.)

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find E or show it doesn't exist

$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$