

2.1.1 (HW2.1#11) Find a matrix B so that $DB = BD$ where $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$B = \begin{bmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{bmatrix} \quad B = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \quad BD = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 5a & 3d & 2g \\ 5b & 3e & 2h \\ 5c & 3f & 2i \end{bmatrix}$$

$$DB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = \begin{bmatrix} 5a & 5d & 5g \\ 3b & 3e & 3h \\ 2c & 2f & 2i \end{bmatrix} \quad \text{so } \begin{cases} a \text{ is free} \\ b = 0 \\ c = 0 \\ d = 0 \end{cases} \begin{cases} e \text{ is free} \\ f = 0 \\ g = 0 \\ h = 0 \\ i \text{ is free} \end{cases}$$

2.1.2 (HW2.1#10) Find a matrix X so that $AX = 0$ where $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

$$X = \begin{bmatrix} 2x_2 & 2x_4 \\ x_2 & x_4 \end{bmatrix} \quad X = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} \quad AX = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} 3x_1 - 6x_2 & 3x_3 - 6x_4 \\ -2x_1 + 4x_2 & -2x_3 + 4x_4 \end{bmatrix}$$

$$\begin{cases} 3x_1 = 6x_2 \\ 4x_2 = 2x_1 \end{cases} \Rightarrow \begin{cases} x_1 = 2x_2 \\ x_2 \text{ is free} \end{cases} \quad \begin{cases} 3x_3 = 6x_4 \\ 4x_4 = 2x_3 \end{cases} \Rightarrow \begin{cases} x_3 = 2x_4 \\ x_4 \text{ is free} \end{cases}$$

2.2.1 Can there be a matrix E such that $EA = I$? (If so, find E ; if not explain why not.)

If $EA = I$, then $EAX = E(AX) = EO = 0$
and $EAX = (EA)X = IX = X$

but $X \neq 0$ (just take $x_2 = 1$
 $x_4 = 1$)

2.2.2 Can there be a matrix E such that $ED = I$? (If so, find E ; if not explain why not.)

$$E = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \quad ED = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 5a & 3d & 2g \\ 5b & 3e & 2h \\ 5c & 3f & 2i \end{bmatrix}$$

$$E = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \quad \text{so } \begin{cases} 5a = 1 \\ 5b = 0 \\ 5c = 0 \end{cases} \begin{cases} a = \frac{1}{5} \\ b = 0 \\ c = 0 \end{cases} \begin{cases} g = 0 \\ h = 0 \\ i = \frac{1}{2} \end{cases}$$

$$\begin{cases} d = 0 \\ e = \frac{1}{3} \\ f = 0 \end{cases}$$