MA322-001 Feb 17 Quiz Name: Key

2.3.1 (HW2.3#3/13) Is the matrix $\begin{bmatrix} -3 & 5 & 8 \\ 0 & -4 & -3 \\ 0 & 0 & 3 \end{bmatrix}$ invertible?

Yes, there is a pivot in each column.

2.3.2 (HW2.3#5) Is the matrix $\begin{bmatrix} 3 & 0 & -3 \\ 2 & 0 & 4 \\ -4 & 0 & 7 \end{bmatrix}$ invertible?

No, because the columns are linearly dependent.

$v_2 = 0 \cdot v_1$

3.3.1 What is the volume of the parallelepiped spanned by the rows of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 10 & 20 \\ 5 & 10 & 20 \end{bmatrix}$?

$R_2 - 4R_1$, $R_3 - 5R_1$: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 5 \end{bmatrix}$ $\xrightarrow[R_2 - \frac{8}{5}R_3]{R_1 - R_2 + R_3}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

The matrix is a rectangular prism with length 1, width 2 and height 5.

Area $= 1 \cdot 2 \cdot 5 = 10$

3.3.2 What is the area of the triangle with vertices $(1,2)$, $(5,3)$, and $(6,8)$?

$\vec{v} = (5,3) - (1,2) = (4,1)$

$\vec{w} = (6,8) - (1,2) = (5,6)$

Area of parallelogram $= \vec{v} \wedge \vec{w} = \det \begin{pmatrix} 4 & 1 \\ 5 & 6 \end{pmatrix} = 4 \cdot 6 - 1 \cdot 5$

$= 24 - 5 = 19$

Area of triangle $= 19/2$