MA322-001 Mar 10 Review

1. Definition of vector space, subspace, and linear transformation.

(a) Set $V = \{p(t) : p \text{ is a quadratic polynomial}\}$. So $t^2 + 3t + 7$ and $-8t^2 + 5$ are in V, but $t - 3$ and $\sin(t)$ are not in V. Is V a vector space?

(b) Set $U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : 2x + 3y = 5 \right\}$. So $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ are in U, but $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ are not in U. Is U a subspace of $\mathbb{R}^2$?

(c) Set $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy = 0 \right\}$. So $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$ are in W, but $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ \sqrt{2} \end{bmatrix}$ are not in W. Is W a subspace of $\mathbb{R}^2$?

Notation: $\mathbb{R}^2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \text{ are any numbers} \right\}$ is the vector space containing all column vectors of length 2. Many people think of it as the Euclidean plane.

2. Linear transformations (Make sure to justify your answers.)

(a) What are the two main requirements for a function to be a linear transformation?

(b) Set $T : \mathbb{R}^2 \to \mathbb{R}$ to be the function with $T\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) = 2x + 3y$. So $T\left(\left[\begin{smallmatrix} \sqrt{2} \\ 7 \end{smallmatrix}\right]\right) = 2\sqrt{2} + 3(7) = 2\sqrt{2} + 21$. Is T a linear transformation?

(c) Set $S : \mathbb{R}^2 \to \mathbb{R}$ to be the function with $S\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) = xy$. So $S\left(\left[\begin{smallmatrix} \sqrt{2} \\ 7 \end{smallmatrix}\right]\right) = \sqrt{2}(7) = 7\sqrt{2}$. Is S a linear transformation?

3. Null spaces (Make sure to justify your answers.)

(a) Set $A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & -6 & 0 \end{bmatrix}$ and $\vec{\mathbf{x}} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Is $\vec{\mathbf{x}}$ in the null space of A?

(b) List 5 vectors from the null space of (that same) A.

(c) Give a basis for the null space of (that same) A.

(d) What is the dimension of the null space of (that same) A?

(e) Give an example of a different matrix B that has the same null space as (that same) A.

4. Column spaces (Make sure to justify your answers.)

(a) Set $A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & -6 & 0 \\ 1 & -3 & 0 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Is $\vec{\mathbf{b}}$ in the column space of A?

(b) List 5 vectors from the column space of (that same) A.

(c) Give a basis for the column space of (that same) A.

(d) What is the dimension of the column space of (that same) A?

(e) Give an example of another matrix C that has the same column space as (that same) A.

I'll probably have A be the same matrix on pages 3 and 4, but the A from class (the one page 3) is not a good exam question on page 4.