

(5.2a) What is the characteristic polynomial of $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$?

$$\det\left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\right) = \det\left(\begin{bmatrix} -c & 1 \\ 1 & 1-c \end{bmatrix}\right) = (-c)(1-c) - 1$$

$$= c^2 - c - 1$$

(5.2b) What are the eigenpairs of A ?

$$\left[\begin{array}{cc|c} -c & 1 & 0 \\ 1 & 1-c & 0 \end{array}\right] \xrightarrow{R_2 \cdot c + R_1} \left[\begin{array}{cc|c} -c & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \begin{cases} x_2 = cx_1 \\ x_1 \text{ is free} \end{cases} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ c \end{bmatrix}$$

$\uparrow$
 $c(1-c) + 1 = -(c^2 - c - 1) = 0$

so $\left(c, \begin{bmatrix} 1 \\ c \end{bmatrix}\right)$ for $c = \frac{1+\sqrt{5}}{2}$ and $c = \frac{1-\sqrt{5}}{2}$

(5.2c) What is the characteristic polynomial of $B = \begin{bmatrix} 7 & 3 & 14 & 21 \\ 0 & 11 & 17 & 5 \\ 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 8 \end{bmatrix}$?

$(7-c)(11-c)(6-c)(8-c)$ (works for any upper triangular matrix)

(5.2d) Give a matrix with no real eigenvalues. Hint: What is the characteristic polynomial

of $\begin{bmatrix} 0 & a \\ 1 & 0 \end{bmatrix}$? Hint: $\det\left(\begin{bmatrix} -c & a \\ 1 & -c \end{bmatrix}\right) = (-c)^2 - a = c^2 - a$

This has no real roots if and only if $a < 0$

so for instance $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$