MA322-001 Apr $2~{\rm Exam}$

Name:_____

- 1. Eigenvectors
- (a) Give the definition of eigenvector.

(b) Is
$$\begin{bmatrix} 1\\2\\3 \end{bmatrix}$$
 an eigenvector of $\begin{bmatrix} 9 & 3 & -4\\0 & 6 & -2\\0 & 0 & 3 \end{bmatrix}$? Explain why or why not.

(c) Why do only square matrices have eigenvectors?

(d) Is
$$\begin{bmatrix} 0\\0\\1 \end{bmatrix}$$
 an eigenvector of $\begin{bmatrix} 9 & 3 & -4\\0 & 6 & -2\\0 & 0 & 3 \end{bmatrix}$? Explain why or why not.

2. Eigenvalues:

(a) Give the definition of eigenvalue.

(b) Is
$$-4$$
 an eigenvalue of $\begin{bmatrix} 9 & 3 & -4 \\ 0 & 6 & -2 \\ 0 & 0 & 3 \end{bmatrix}$? Explain why or why not.

(c) Is 9 an eigenvalue of
$$\begin{bmatrix} 9 & 3 & -4 \\ 0 & 6 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$
? Explain why or why not.

(d) Give an example of a matrix with eigenvalue 5. Explain why your example works.

3. Diagonal matrices

On this page,
$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$
, $D = \begin{bmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{bmatrix}$, and $\vec{\mathbf{v}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.
(a) Calculate $A^7 - 5A^2 + A$

(b) Calculate $(A + D)^2$

(c) Calculate $(D^5 - 7D^3)\vec{\mathbf{v}}$

(d) What are the eigenpairs of A + D?

4. General matrices

On this page,
$$A = \begin{bmatrix} 5 & 0 & -4 \\ 1 & 2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$$
, and it has
eigenpairs $(c_1, \vec{\mathbf{v}}_1) = \left(1, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right), (c_2, \vec{\mathbf{v}}_2) = \left(2, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$, and $(c_3, \vec{\mathbf{v}}_3) = \left(3, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right)$.
The vector $\vec{\mathbf{w}} = \begin{bmatrix} 4 \\ 5 \\ 4 \end{bmatrix}$ and it can be written as $4\vec{\mathbf{v}}_1 + 5\vec{\mathbf{v}}_2$.
(a) Calculate $(A^7 - 5A^2 + A)\vec{\mathbf{v}}_1$

(b) Calculate
$$(A^7 - 5A^2 + A)\vec{\mathbf{v}}_2$$

(c) Calculate $(A^7 - 5A^2 + A)\vec{\mathbf{w}}$

(d) What are the eigenpairs of A^2 ?