MA322-001 Apr 7 Quiz Name: Key

Let $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

1. Compute the following inner-products for $\vec{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

(a) $\vec{v} \cdot \vec{e}_1$ $= 4 \cdot 1 + 5 \cdot 0 + 6 \cdot 0 = \boxed{4}$

(b) $\vec{v} \cdot \vec{e}_2$ $= 4 \cdot 0 + 5 \cdot 1 + 6 \cdot 0 = \boxed{5}$

(c) $\vec{v} \cdot \vec{e}_3$ $= 4 \cdot 0 + 5 \cdot 0 + 6 \cdot 1 = \boxed{6}$

Let $\vec{f}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{f}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{f}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

2. Compute the following inner-products for $\vec{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

(a) $\vec{v} \cdot \vec{f}_1$ $= 4 \cdot 1 + 5 \cdot 0 + 6 \cdot 0 = \boxed{4}$

(b) $\vec{v} \cdot \vec{f}_2$ $= 4 \cdot 1 + 5 \cdot 1 + 6 \cdot 0 = 4 + 5 = \boxed{9}$

(c) $\vec{v} \cdot \vec{f}_3$ $= 4 \cdot 1 + 5 \cdot 1 + 6 \cdot 1 = 4 + 5 + 6 = \boxed{15}$

Let $\vec{g}_1 \cdot \vec{g}_2 = 0$ and $\|\vec{g}_1\| = \|\vec{g}_2\| = 1$.

3. Compute the following inner products for $\vec{v} = 4\vec{g}_1 + 5\vec{g}_2$

(a) $\vec{v} \cdot \vec{g}_1$ $= \langle 4\vec{g}_1 + 5\vec{g}_2, \vec{g}_1 \rangle = 4\langle \vec{g}_1, \vec{g}_1 \rangle + 5\langle \vec{g}_2, \vec{g}_1 \rangle$

$= 4\|\vec{g}_1\|^2 + 5 \cdot 0 = \boxed{4}$

(b) $\vec{v} \cdot \vec{g}_2$ $= \langle 4\vec{g}_1 + 5g_2, g_2 \rangle = 4\langle \vec{g}_1, \vec{g}_2 \rangle + 5\langle g_2, g_2 \rangle$

$= 4 \cdot 0 + 5\|g_2\|^2 = \boxed{5}$

4. What is the lesson?

If the basis is orthogonal, inner products let you read off the coefficients of a vector written as a linear combination of those basis vectors.