MA322-001 Apr 16 Exam Name:________________________

1. Inner products and orthogonality - Show work clearly.

(a) Compute the dot product of $\vec{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{\mathbf{a}}_2 = \begin{bmatrix} 100 \\ 10 \\ -1 \end{bmatrix}$

(b) Suppose $\vec{\mathbf{b}}_1 \cdot \vec{\mathbf{b}}_1 = 1000000$, $\vec{\mathbf{b}}_1 \cdot \vec{\mathbf{b}}_2 = \vec{\mathbf{b}}_2 \cdot \vec{\mathbf{b}}_1 = 1000$, and $\vec{\mathbf{b}}_2 \cdot \vec{\mathbf{b}}_2 = 1$. Compute $(1\vec{\mathbf{b}}_1 + 3\vec{\mathbf{b}}_2) \cdot (5\vec{\mathbf{b}}_1 + 7\vec{\mathbf{b}}_2)$.

(c) Suppose $\vec{\mathbf{c}}_1 \cdot \vec{\mathbf{c}}_1 = 1$, $\vec{\mathbf{c}}_1 \cdot \vec{\mathbf{c}}_2 = 0$, and $\vec{\mathbf{c}}_2 \cdot \vec{\mathbf{c}}_2 = 1$. Compute $(1\vec{\mathbf{c}}_1 + 3\vec{\mathbf{c}}_2) \cdot (5\vec{\mathbf{c}}_1 + 7\vec{\mathbf{c}}_2)$.

(d) Suppose $\|\vec{\mathbf{d}}_1\| = 1$, $\vec{\mathbf{d}}_1 \cdot \vec{\mathbf{d}}_2 = 0$, and $\|\vec{\mathbf{d}}_2\| = 1$. Compute $\|8\vec{\mathbf{d}}_1 + 15\vec{\mathbf{d}}_2\|$.

(e) Give an example of two vectors $\vec{\mathbf{f}}_1$ and $\vec{\mathbf{f}}_2$ with no 0s in their coordinates with $\vec{\mathbf{f}}_1 \cdot \vec{\mathbf{f}}_2 = 0$.

2. Projections - Show work clearly.

(a) Let $\vec{\mathbf{a}}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\vec{\mathbf{a}}_2 = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$. Compute the projection of $\vec{\mathbf{a}}_2$ onto $\vec{\mathbf{a}}_1$.

(b) Let $\vec{\mathbf{b}}_1 \cdot \vec{\mathbf{b}}_1 = 2$, $\vec{\mathbf{b}}_1 \cdot \vec{\mathbf{b}}_2 = 14$, and $\vec{\mathbf{b}}_2 \cdot \vec{\mathbf{b}}_2 = 23$. Compute the projection of $\vec{\mathbf{b}}_2$ onto $\vec{\mathbf{b}}_1$.

(c) Let $\vec{\mathbf{c}}_1 \cdot \vec{\mathbf{c}}_1 = 2$, $\vec{\mathbf{c}}_1 \cdot \vec{\mathbf{c}}_2 = 0$, and $\vec{\mathbf{c}}_2 \cdot \vec{\mathbf{c}}_2 = 23$. Compute the projection of $\vec{\mathbf{c}}_2$ onto $\vec{\mathbf{c}}_1$.

(d) Let $\vec{\mathbf{d}} = \begin{bmatrix} 2 \\ 14 \\ 23 \end{bmatrix}$. Compute the projection of $\vec{\mathbf{d}}$ onto $\vec{\mathbf{d}}$.

(e) Let $\vec{\mathbf{f}}_1 = \begin{bmatrix} 2 \\ 14 \\ 23 \end{bmatrix}$ and $\vec{\mathbf{f}}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Compute the projection of $\vec{\mathbf{f}}_2$ onto $\vec{\mathbf{f}}_1$.

3. Gram-Schmidt - Show work clearly.

(a) Let $\vec{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{\mathbf{a}}_2 = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, $\vec{\mathbf{a}}_3 = \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \end{bmatrix}$. Compute vectors $\vec{\mathbf{g}}_1$, $\vec{\mathbf{g}}_2$, and $\vec{\mathbf{g}}_3$ that are orthogonal and span the same subspace as $\vec{\mathbf{a}}_1$, $\vec{\mathbf{a}}_2$, and $\vec{\mathbf{a}}_3$ using the Gram-Schmidt procedure.

If you can do this quickly, you can tell people you invented the insta-Gram-Schmidt

4. Least Squares - Show work clearly.

(a) Suppose $\vec{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\vec{\mathbf{a}}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{\mathbf{a}}_3 = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix}$. Notice that $\vec{\mathbf{a}}_1 \cdot \vec{\mathbf{a}}_1 = 10$, $\vec{\mathbf{a}}_1 \cdot \vec{\mathbf{a}}_2 = 0$, $\vec{\mathbf{a}}_1 \cdot \vec{\mathbf{a}}_3 = 20$, $\vec{\mathbf{a}}_2 \cdot \vec{\mathbf{a}}_2 = 1$, and $\vec{\mathbf{a}}_2 \cdot \vec{\mathbf{a}}_3 = 6$. Find y_1 and y_2 so that $y_1\vec{\mathbf{a}}_1 + y_2\vec{\mathbf{a}}_2$ is as close as possible to $\vec{\mathbf{a}}_3$.

(b) How close can $y_1\vec{\mathbf{a}}_1 + y_2\vec{\mathbf{a}}_2$ be to $\vec{\mathbf{a}}_3$?

(c) $\vec{\mathbf{c}}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{\mathbf{c}}_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{\mathbf{c}}_3 = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix}$. How close can $x_1\vec{\mathbf{c}}_1 + x_2\vec{\mathbf{c}}_2$ be to $\vec{\mathbf{c}}_3$?

Bonus: What x_1 and x_2 work to get that closest value? Hint: $\vec{\mathbf{c}}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 2\vec{\mathbf{c}}_1$.