A population of rabbits has three basic age groups: babies, adults, and seniors.

The populations satisfy a simple recurrence:

$$\begin{cases} b_k = a_{k-1} + 0.5 s_{k-1} & \text{Each adult pair has a pair of babies, each senior pair has a single baby} \\ a_k = 0.9 b_{k-1} & \text{90\% of babies survive to be adults} \\ s_k = 0.7 a_{k-1} & \text{70\% of adults survive to be seniors} \end{cases}$$

Main question: By what percentage does the rabbit population increase (or decrease) each season? How many adult rabbits will there be (compared to the number of babies)? How many seniors per baby?

Solution: Let $\vec{r}_k = \begin{bmatrix} b_k \\ a_k \\ s_k \end{bmatrix}$ measure the population of rabbits. Notice that for $A = \begin{bmatrix} 0.0 & 1.0 & 0.5 \\ 0.9 & 0.0 & 0.0 \\ 0.0 & 0.7 & 0.0 \end{bmatrix}$ we have $\vec{r}_k = A\vec{r}_{k-1}$.

We use a standard tool to find its eigenpairs $(c_i, \vec{w}_i)$, and get

$$\left(1.0904, \begin{bmatrix} 1 \\ 0.8254 \\ 0.5299 \end{bmatrix}\right), \quad \left(-0.6364, \begin{bmatrix} 1 \\ -1.4141 \\ 1.5553 \end{bmatrix}\right), \quad \left(-0.4539, \begin{bmatrix} 1 \\ -1.9827 \\ 3.0576 \end{bmatrix}\right)$$

Whatever the starting population $\vec{v}_0$, it can be written as a linear combination

$$\vec{v}_0 = a_1\vec{w}_1 + a_2\vec{w}_2 + a_3\vec{w}_3,$$

and then the ending population

$$\vec{v}_k = A^k\vec{v}_0 = a_1 c_1^k \vec{w}_1 + a_2 c_2^k \vec{w}_2 + a_3 c_3^k \vec{w}_3$$

Note that $0.001 \geq |c_2|^k \approx |c_3|^k \approx 0$ for $k \geq 10$, so it doesn't take very long for a_2 and a_3 to be more than 1000 times less important than a_1, so that $\vec{v}_k \approx a_1 c_1^k \vec{w}_1$, a quantity that grows at the rate of 9.04% ($c_1 - 1$ expressed as a percent). For every baby rabbit, we only have 0.8254 adult rabbits, and only 0.5299 senior rabbits (these are just $\vec{w}_1$), so that the population is skewed towards the young. The population doubles every 8.0091 years ($\log(2)/\log(c_1)$).