Small number version.

1. Give standard representations for each arithmetic problem (preferably without a calculator):

$5^1 \pmod{23}$ 5

$5^2 = 5 \times 5 \pmod{23}$ $25 \equiv 2$

$5^4 = (5^2) \times (5^2) \pmod{23}$ $2 \times 2 = 4$

$5^8 = (5^4) \times (5^4) \pmod{23}$ $4 \times 4 = 16 \equiv -7$

$5^{16} = (5^8) \times (5^8) \pmod{23}$ $(-7) \times (-7) = 49 \equiv 49 - 46 = 3$

2. (a) Choose a SECRET number A between 2 and 21 (numbers smaller than 2 are too easy, numbers bigger than 21 are repeats). $A = 20$

(b) SECRETLY calculate the standard representative $5^A \pmod{23}$ using your answers from part 1. For example if $A = 13 = 8 + 4 + 1$ then $5^A = (5^8) \times (5^4) \times (5^1) \equiv 21 \pmod{23}$.

$5^A = 5^{16} \times 5^4 = 3 \times 4 = 12$

(c) Tell your partner your final answer from part (b), for example "21". Write down what they tell you here: 7

(d) Now take the number c your partner gave you and raise it to the Ath power too, $c^A \pmod{23}$. Use the same trick, $c^{13} = c^8 \times c^4 \times c^1$ and $c^8 = c^4 \times c^4$ and $c^4 = c^2 \times c^2$.

$7^{20} = 7^{16} \times 7^4 = 6 \times 9 = 54 \equiv 54 - 46 = 8$

$7^1 = 7$
$7^2 = 3$
$7^4 = 9$
$7^8 = 81 \equiv 12$
$7^{16} = 6$

(e) Now on the count of 3, say your number together!

We both said 8

Larger number version.

3. Give standard representations for each arithmetic problem (preferably without a calculator):

$2^1 \pmod{101}$ 2

$2^2 = 2 \times 2 \pmod{101}$ 4

$2^4 = (2 \times 2) \times (2 \times 2) \pmod{101}$ 16

$2^8 = (2^4) \times (2^4) \pmod{101}$ $256 \equiv 256 - 202 = 54$

$2^{10} = (2^8) \times (2^2) \pmod{101}$ $54 \times 4 = 216 \equiv 216 - 202 = 14$

$2^{16} = (2^8) \times (2^8) \pmod{101}$ $\begin{array}{r} 54 \\ \times\ 54 \\ \hline 216 \\ 270 \end{array} = 2916 \equiv 2916 - 2929 = -13 \equiv 88$

$2^{32} = (2^{16}) \times (2^{16}) \pmod{101}$ ... the answer is 68 in case you run out of time

$2^{64} = (2^{32}) \times (2^{32}) \pmod{101}$ ... the answer is 79 in case you run out of time

4. (a) Choose a SECRET number A between 7 and 100 (numbers smaller than 7 are too easy, numbers bigger than 100 are repeats). 9

(b) SECRETLY calculate the standard representative $2^A \pmod{101}$ using your answers from part 1. For example if $A = 13 = 8 + 4 + 1$ then $2^A = (2^8) \times (2^4) \times (2^1) \equiv 11 \pmod{101}$.

$2^9 = 2^8 \times 2^1 = 54 \times 2 = 108 \equiv 7$

(c) Tell your partner your final answer from part (b), for example "21". Write down what they tell you here: 21

(d) Now take the number c your partner gave you and SECRETLY raise it to the Ath power too, $c^A \pmod{101}$. Use the same trick, $c^{13} = c^8 \times c^4 \times c^1$ and $c^8 = c^4 \times c^4$ and $c^4 = c^2 \times c^2$.

$21^9 = 21^8 \times 21^1 = 5 \times 21 = 105 \equiv 4$

$21^1 = 21$
$21^2 = 37$
$21^4 = 56$
$21^8 = 5$

(e) Now on the count of 3, say your number together!

4, we both said 4.