Small number version.

1. Give standard representations for each arithmetic problem (preferably without a calculator):

$5^1 \pmod{23}$

$5^2 = 5 \times 5 \pmod{23}$

$5^4 = (5^2) \times (5^2) \pmod{23}$

$5^8 = (5^4) \times (5^4) \pmod{23}$

$5^{16} = (5^8) \times (5^8) \pmod{23}$

2. (a) Choose a SECRET number $A$ between 2 and 21 (numbers smaller than 2 are too easy, numbers bigger than 21 are repeats).

(b) SECRETLY calculate the standard representative $5^A \pmod{23}$ using your answers from part 1. For example if $A = 13 = 8 + 4 + 1$ then $5^A = (5^8) \times (5^4) \times (5^1) \equiv 21 \pmod{23}$.

(c) Tell your partner your final answer from part (b), for example "21". Write down what they tell you here:

(d) Now take the number $c$ your partner gave you and raise it to the $A$th power too, $c^A \pmod{23}$. Use the same trick, $c^{13} = c^8 \times c^4 \times c^1$ and $c^8 = c^4 \times c^4$ and $c^4 = c^2 \times c^2$.

(e) Now on the count of 3, say your number together!

Larger number version.

3. Give standard representations for each arithmetic problem (preferably without a calculator):

$2^1 \pmod{101}$

$2^2 = 2 \times 2 \pmod{101}$

$2^4 = (2 \times 2) \times (2 \times 2) \pmod{101}$

$2^8 = (2^4) \times (2^4) \pmod{101}$

$2^{10} = (2^8) \times (2^2) \pmod{101}$

$2^{16} = (2^8) \times (2^8) \pmod{101}$

$2^{32} = (2^{16}) \times (2^{16}) \pmod{101}$ ... the answer is 68 in case you run out of time

$2^{64} = (2^{32}) \times (2^{32}) \pmod{101}$ ... the answer is 79 in case you run out of time

4. (a) Choose a SECRET number $A$ between 7 and 100 (numbers smaller than 7 are too easy, numbers bigger than 100 are repeats).

(b) SECRETLY calculate the standard representative $2^A \pmod{101}$ using your answers from part 1. For example if $A = 13 = 8 + 4 + 1$ then $2^A = (2^8) \times (2^4) \times (2^1) \equiv 11 \pmod{101}$.

(c) Tell your partner your final answer from part (b), for example "21". Write down what they tell you here:

(d) Now take the number $c$ your partner gave you and SECRETLY raise it to the $A$th power too, $c^A \pmod{101}$. Use the same trick, $c^{13} = c^8 \times c^4 \times c^1$ and $c^8 = c^4 \times c^4$ and $c^4 = c^2 \times c^2$.

(e) Now on the count of 3, say your number together!