Blair, Casey, and Devin are throwing a for-profit party and splitting the money at the end evenly. They all agree live music is probably a good idea, but disagree on how much to pay for it. Since they are splitting the profits evenly, they are only willing to pay a third (each) of what they think the band will bring in.

**Which band should they book and how should they split the bill?**

Here are their guesses on how much each band would bring in, along with the band's cost:

| | Ford Theater Reunion | SHOZO | Big Maracas | No Band |
|---|---|---|---|---|
| Blair | $3000 | $2400 | $2400 | $0 |
| Casey | $3300 | $3000 | $3300 | $0 |
| Devin | $2700 | $3000 | $3300 | $0 |
| Band | $3000 | $2700 | $2700 | $0 |

The band will only be happy if it is paid in full. Blair, Casey, and Devin will only be happy if they pay less than a third of what they think the band will bring in. Can we make everyone happy according to those requirements?

What is wrong with the following solutions?

(a) They hire FTR. Blair pays $800, Casey pays $800, Devin pays $800.

They pay FTR, $800 + $800 + $800 = $2400 not $3000 so FTR either cancels or sues. ☹

(b) They hire FTR. Blair pays $1000, Casey pays $1000, Devin pays $1000.

FTR is happy, but Devin is not. Devin expects to get $\frac{\$2700}{3} = \$900$ but paid $1000. Devin expects to lose $100 on this deal.

(c) They hire SHOZO. Blair pays $900, Casey pays $900, Devin pays $900.

Now Blair is upset. Pays $900, expects to get back $\frac{\$2400}{3} = \$800$, so expects to lose $100.

(d) They hire BigM. Blair pays $900, Casey pays $900, Devin pays $900.

Exactly the same as (c) for Blair (though different/better for Casey and Devin...)

(e) They hire BigM. Blair pays $700, Casey pays $1000, Devin pays $1000.

Everybody is OK! BigM gets $700 + $1000 + $1000 = $2700 ☺
Blair expects to get $\frac{\$2400}{3} = \$800$ by paying $700, so $100 ☺
C+D expect to get $3300/3 = $1100 by paying $1000, so $100 ☺

(f) They hire BigM. Blair pays $720, Casey pays $990, Devin pays $990.

Everybody is OK! BigM still gets $2700
Blair gets $800 - $720 = $80 extra. $80/$720 ≈ 11%
C+D get $1100 - $990 = $110 extra. $110/990 ≈ 11%

How much profit does each person expect to make?

In (a) - (d) someone expects to lose money.
In (e) everybody (well Blair, Casey, Devin) expects to make $100 extra
In (f) everybody (B+C+D) expects to make 11% of what they pay

In part (e) and (f) there is extra money, Surplus.

But it is divided diffently.

In (e) there is $100 + $100 + $100 = $300 split evenly
same!

In (f) there is $80 + $110 + $110 = $300, split proportionally

Which band (or no band) is best?

I would say it is the one where we have the most surplus to split!

How do we find the surplus?
It is the average value minus the cost

| | FTR | 9HOZO | BigM | No Band |
|---|---|---|---|---|
| AVE? | $\frac{3000+3300+2700}{3}$ | $\frac{2400+3000+3000}{3}$ | $\frac{2400+3300+3300}{3}$ | $\frac{0+0+0}{3}$ |
| AVE | = 3000 | = 2700 | = 3000 | = 0 |
| Cost | 3000 | 2700 | 2700 | 0 |
| Surplus | $0 | $0 | $300 | $0 |

same!

Big Maracas gives the opportunity for $300 profit