

## MA111: Contemporary mathematics

Alex and Blair split a 2-bedroom apartment for \$550 per month so that each was paying \$50 less than they expected, given how great their rooms were.

	Master	Other
Alex	\$350	\$325
Blair	\$315	\$300
Total	\$550 total	
Who gets it?	Alex	Blair
Rent	\$300	\$250

Why does Alex think Blair got the better deal?

Can you fix it?

[ Alex should think Alex got a good deal. Alex should think Alex got a better deal than Blair (or the same). Vice Versa for Blair. Rent should be paid in full. ]

Schedule:

- Exam 3 is in class, Thu Nov 13th, 2014

Today we examine the inevitable betrayal.

## While we pass out the worksheet

Alex thinks Alex got a \$350 room for only \$300 rent – \$50 extra 😊

Blair thinks Blair got a \$300 room for only \$250 rent – \$50 extra 😊

Alex thinks Blair got a \$325 room for only \$250 rent – \$75 extra...

- It is not a problem that they disagree, the problem is:

Alex thinks Blair got \$75 off when Alex only got \$50 off

- This is called **envy**.

Alex got \$50 more than Alex deserved, yay! 😊

But Blair got even MORE. Jerky jerk face. 😞

- Can we actually make both people happy?

# Important concepts

- Assignment problem: each person gets one thing;  
maximize total happiness and then use money to even things up.  
hard, but you guys are good at it
- Shared experience problem: should the group do it?  
is the total benefit to the group more than the total cost?  
if so, divide the cost according to how much people enjoy it
- Indivisible items currently shared: who should get the object  
Give each object to whoever wants it most  
Those people pay everyone else for their portion

## Details after you decide who gets what

- 1 Everybody pays the maximum they said they would into a shared pile of money
- 2 The pile is used to pay everyone (the minimum possible if there is a choice)
- 3 The leftover money is split between everyone, either equally or proportional to how much they paid in

## Details for shared object

- Shared object is basically the same

in #1 the only ones who pay are the ones who get an object – they pay full price

in #2 everyone gets  $1/n$  of their price back – even the one who got the objects

step #3 is totally the same

- Many people combine steps #1 and #2:

with 4 people, each person who gets an object pays 75% to the shared pile, and the three people who didn't get it take 25% back.

Even though  $25\% + 25\% + 25\% = 75\%$ , the “25% of what” are smaller than the “75% of what”, so there is money leftover for step #3.

# Exit quiz

- Alex and Blair are trying to decide whether to get Premium Cable
- It costs \$45 per month extra for the premium package
- Alex thinks it would be worth \$40 per month
- Blair thinks it would be worth \$20 per month
- Together they think it is worth \$60 per month, so \$45 is super-cheap!
- How much should each one pay to avoid **envy**, pay the bills, and make each person feel like they got a good deal?