MA322-007 Jan 20 worksheet

Vector geometry!

One trick to getting used to linear algebra is to combine geometric intuition with the number crunching (we've done) and algebraic manipulations (to come). This worksheet is meant first to be read from top to bottom, next solve part (d), then solve (c), then (b), and finally (a). It is also example 4 in the book.

(a) Can you solve this system of equations?

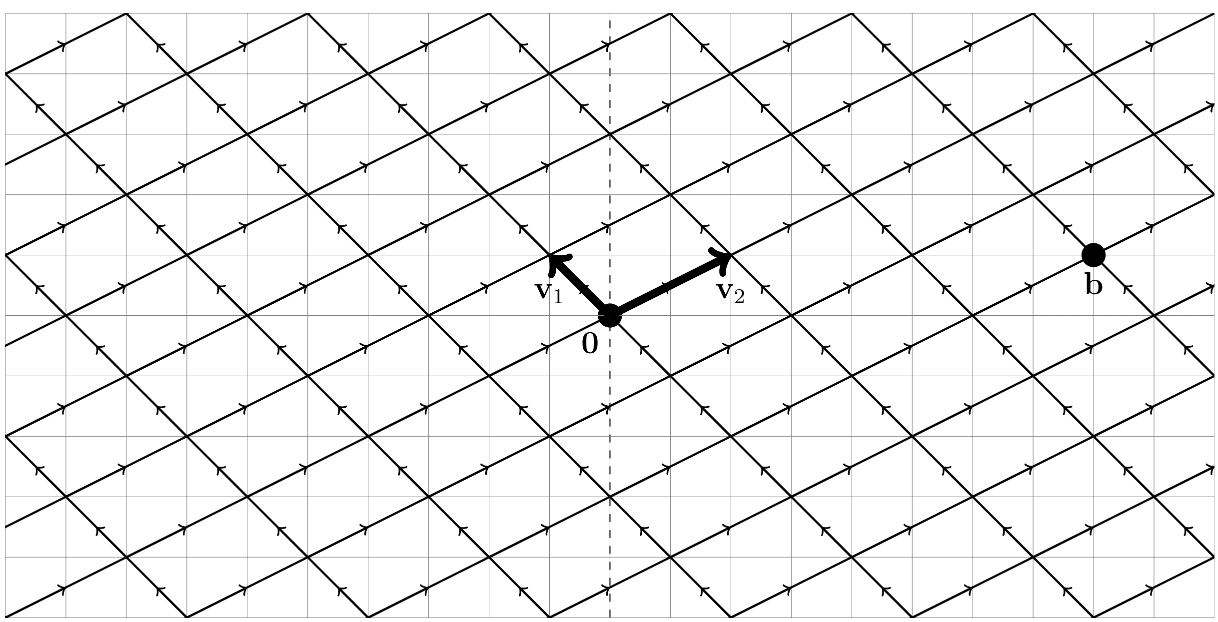
$$\begin{cases} -1x_1 + 2x_2 = 8 \\ 1x_1 + 1x_2 = 1 \end{cases}$$

(b) Can you solve this vector equation?

$$x_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

(c) Can you write b as a linear combination $x_1 v_1 + x_2 v_2 = b$ where $b = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$, $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

(d) Can you write b as a linear combination $x_1 v_1 + x_2 v_2 = b$ using this picture and no calculation other than counting?



Stoichiometry!

This worksheet is meant to be read from top to bottom and the back to the top. The answers are likely to be "no" or "I don't know" on the way down, but should be better on the way back up.

(a) Can you balance this reaction?

$$\underbrace{\qquad\quad} CH_4 + \underbrace{\qquad\quad} CO_2 \to \underbrace{\qquad\quad} H_2O$$

(b) Can you solve this vector equation?

$$x_1 \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

(c) Can you solve this system of equations?

$$\{x_1 + x_2 = 0,\ 4x_1 = 2x_3,\ 2x_2 = x_3\}$$

(d) Can you row reduce this matrix?

$$\begin{array}{c} \begin{array}{ccc|c} x_1 & x_2 & x_3 & \# \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 4 & 0 & -2 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right] \end{array}$$

(e) What is the general solution to the system of equations with this augmented matrix?

$$\begin{array}{c} \begin{array}{ccc|c} x_1 & x_2 & x_3 & \# \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

(f) Now go back and answer (c), (b), and (a).

Harder Stoichiometry. This problem was from a chemistry class with a MA322 pre-requisite.

Read me: Convince yourself that solving (a) is the same as solving (b), which is the same as solving (c), and that (d) would be all the actual calculation, so that the answer to (e) is very useful, but (f) is good enough. Now answer (e), (f) and (a).

(a) Balance this reaction

$$\underbrace{\qquad} P_2I_4 + \underbrace{\qquad} P_4 + \underbrace{\qquad} H_2O \to \underbrace{\qquad} PH_4I + \underbrace{\qquad} H_3PO_4$$

(b) Solve this vector equation:

$$x_1 \begin{bmatrix} 0 \\ 4 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = x_4 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 3 \\ 0 \\ 4 \\ 1 \end{bmatrix} \quad \begin{matrix} H \\ I \\ O \\ P \end{matrix}$$

(c) Solve this system of equations:

$$\begin{cases} H & 2x_3 = 4x_4 + 3x_5 \\ I & 4x_1 = x_4 \\ O & x_3 = 4x_5 \\ P & 2x_1 + 4x_2 = x_4 + x_5 \end{cases}$$

(d) Show that the matrix A is row equivalent to the matrix B (personally, I'd ask a computer to do this; you could just believe me that it is true during class, and check it at home in a spreadsheet).

$$A = \begin{bmatrix} 0 & 0 & 2 & -4 & -3 & 0 \\ 4 & 0 & 0 & -1 & -0 & 0 \\ 0 & 0 & 1 & -0 & -4 & 0 \\ 2 & 4 & 0 & -1 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 & -10/32 & 0 \\ 0 & 1 & 0 & 0 & -13/32 & 0 \\ 0 & 0 & 1 & 0 & -128/32 & 0 \\ 0 & 0 & 0 & 1 & -40/32 & 0 \end{bmatrix}$$

(e) Describe all possible solutions to the system of equations whose augmented matrix is B.

(f) What is a solution in which all the variables are integers?

MA322-007 Jan 20 quiz Name:________________________

(HW1.1#18) How would you determine where the three planes $2x+4y+4z=4$, $y-2z=-2$, $2x+3y=0$ intersect?

(HW1.2#13) Describe the general solution to the system of equations represented by the following augmented matrix. Make sure your solution has no circular or nested definitions (free variables should be labelled as free, basic variables should be defined in terms of free variables and not in terms of other basic variables; look these words up in 1.2 if you don't know what they mean).

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \# \end{array} \\ \left[\begin{array}{rrrrr|r} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \left\{\begin{array}{l} x_1 \underline{\hspace{6cm}} \\ x_2 \underline{\hspace{6cm}} \\ x_3 \underline{\hspace{6cm}} \\ x_4 \underline{\hspace{6cm}} \\ x_5 \underline{\hspace{6cm}} \end{array}\right.$$

1.3.1 Write $b = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

1.6.1 Balance this equation: $\underbrace{\hspace{2cm}} C_3H_8 + \underbrace{\hspace{2cm}} O_2 \rightarrow \underbrace{\hspace{2cm}} CO_2 + \underbrace{\hspace{2cm}} H_2O$

$$\left[\begin{array}{rrrr|r} 3 & 0 & -1 & 0 & 0 \\ 8 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow{R_1=3R_1-R_2} \left[\begin{array}{rrrr|r} 1 & 0 & -3 & 2 & 0 \\ 8 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow{R_2=R_2-8R_1} \left[\begin{array}{rrrr|r} 1 & 0 & -3 & 2 & 0 \\ 0 & 0 & 24 & -18 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow[R_2/2, R_3/24]{R_3 \leftrightarrow R_2} \left[\begin{array}{rrrr|r} 1 & 0 & -3 & 2 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right] \xrightarrow[R_2=R_2+R_3]{R_1=R_1+3R_3} \left[\begin{array}{rrrr|r} 1 & 0 & 0 & -1/4 & 0 \\ 0 & 1 & 0 & -5/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right]$$