MA322-007 Jan 22 quiz Name:______________

HW1.3(#9) Write this system of equations as a single equation of vectors:

$$\begin{aligned} x_2 + 5x_3 &= 0 \\ 4x_1 + 6x_2 - x_3 &= 0 \\ -x_1 + 3x_2 + 8x_3 &= 0 \end{aligned}$$

$$X_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + X_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + X_3 \begin{bmatrix} 5 \\ -1 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

HW1.3(#13) Write $\vec{\mathbf{b}}$ as a linear combination of the columns of A where
$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}$, $\vec{\mathbf{b}} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$. Trick Question (Sorry ☺)

$$R_2 + 2R_1 \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$\rightarrow 0x_1 + 0x_2 + 0x_3 = 3$ is impossible.

b is <u>not</u> in the span of the columns of A

1.4a Write HW1.3#9 as a single equation in the form $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

$$\begin{bmatrix} 0 & 1 & 5 \\ 4 & 6 & -1 \\ -1 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.4b(HW1.3#22) Give an example of a matrix A and a vector $\vec{\mathbf{b}}$ so that the equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has no solution.

This one works ☺

Stoichiometry!

(a) Balance this reaction: $\underbrace{1}\,C_3H_8 + \underbrace{5}\,O_2 \to \underbrace{3}\,CO_2 + \underbrace{4}\,H_2O$

From (b) or (f)

(b) Find scalars x_1, x_2, x_3, and x_4 (not all zero) so that

$$\underbrace{x_1}_{}^{1}\begin{bmatrix}3\\8\\0\end{bmatrix} + \underbrace{x_2}^{5}\begin{bmatrix}0\\0\\2\end{bmatrix} = \underbrace{x_3}^{3}\begin{bmatrix}1\\0\\2\end{bmatrix} + \underbrace{x_4}^{4}\begin{bmatrix}0\\2\\1\end{bmatrix}$$

From (f)

(c) Solve the equations From (e)

$$\begin{cases} 3x_1 = x_3 \\ 8x_2 = 2x_4 \\ 2x_2 = 2x_3 + x_4 \end{cases}$$

$X_1 = \frac{1}{4} X_4$
$X_2 = \frac{5}{4} X_4$
$X_3 = \frac{3}{4} X_4$
X_4 is free

(d) Row reduce this matrix:

$$\left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

Hint: $\left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow[R_2-(8/3)R_1]{R_1/3,R_3/2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 8/3 & -2 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \end{array}\right] \xrightarrow[R_2\leftrightarrow R_3]{(3/8)R_2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right] \xrightarrow[R_2+R_3]{R_1+(1/3)R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/4 & 0 \\ 0 & 1 & 0 & -5/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right]$

$R_2 - 8R_1$

1

$X_1 - \frac{1}{4} X_4 = 0$
$X_2 - \frac{5}{4} X_4 = 0$
$X_3 - \frac{3}{4} X_4 = 0$

(e) What is the general solution?

2

$X_1 = \frac{1}{4} X_4$
$X_2 = \frac{5}{4} X_4$
$X_3 = \frac{3}{4} X_4$
X_4 is free

(f) What is a non-zero solution with all integers? Set $X_4 = 4$

3

$X_1 = 1$ $X_3 = 3$
$X_2 = 5$ $X_4 = 4$