

HW1.3(#9) Write this system of equations as a single equation of vectors: $x_2 + 5x_3 = 0$
 $4x_1 + 6x_2 - x_3 = 0$
 $-x_1 + 3x_2 + 8x_3 = 0$

$$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

HW1.3(#13) Write $\vec{b}$ as a linear combination of the columns of A where

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}. \quad \text{Trick Question (Sorry :)}$$

$$R_2 + 2R_1 \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array} \right]$$

$\rightarrow 0x_1 + 0x_2 + 0x_3 = 3$ is impossible.

$\vec{b}$ is not in the span of the columns of A

1.4a Write HW1.3#9 as a single equation in the form $A\vec{x} = \vec{b}$.

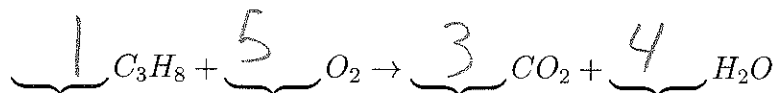
$$\begin{bmatrix} 0 & 1 & 5 \\ 4 & 6 & -1 \\ -1 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.4b(HW1.3#22) Give an example of a matrix A and a vector $\vec{b}$ so that the equation $A\vec{x} = \vec{b}$ has no solution.

This one works :)

Stoichiometry!

(a) Balance this reaction:



From (b) or (f)

(b) Find scalars x_1, x_2, x_3 , and x_4 (not all zero) so that

$$\underbrace{1}_{x_1} \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + \underbrace{5}_{x_2} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \underbrace{3}_{x_3} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + \underbrace{4}_{x_4} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

From (f)

(c) Solve the equations

From (e)

$$\begin{cases} 3x_1 = x_3 \\ 8x_2 = 2x_4 \\ 2x_2 = 2x_3 + x_4 \end{cases}$$

$$x_1 = \frac{1}{4} x_4$$

$$x_2 = \frac{5}{4} x_4$$

$$x_3 = \frac{3}{4} x_4$$

x_4 is free

(d) Row reduce this matrix:

$$\left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array} \right]$$

$$\text{Hint: } \left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array} \right] \xrightarrow[R_2 - (8/3)R_1]{R_1/3, R_3/2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 8/3 & -2 & 0 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \end{array} \right] \xrightarrow[R_2 \leftrightarrow R_3]{(3/8)R_2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array} \right] \xrightarrow[R_2 + R_3]{R_1 + (1/3)R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/4 & 0 \\ 0 & 1 & 0 & -5/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array} \right]$$

①

$$\begin{aligned} x_1 - \frac{1}{4} x_4 &= 0 \\ x_2 - \frac{5}{4} x_4 &= 0 \\ x_3 - \frac{3}{4} x_4 &= 0 \end{aligned}$$

(e) What is the general solution?

②

$$x_1 = \frac{1}{4} x_4$$

$$x_2 = \frac{5}{4} x_4$$

$$x_3 = \frac{3}{4} x_4$$

x_4 is free

③

(f) What is a non-zero solution with all integers?

Set $x_4 = 4$

$$x_1 = 1$$

$$x_3 = 3$$

$$x_2 = 5$$

$$x_4 = 4$$