MA322-007 Jan 29 Practice Exam Name:________________________

1. Question conversion. (You don't need to solve any of the linear systems on this page.

(a) Convert this system of equations to an augmented matrix $[A|b]$

$$\left\{\begin{array}{r} x_1 + 4x_2 + 7x_3 = 11 \\ 2x_1 + 5x_2 + 8x_3 = 13 \\ 6x_1 + 9x_2 + 3x_3 = 15 \end{array}\right\}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 7 & 11 \\ 2 & 5 & 8 & 13 \\ 6 & 9 & 3 & 15 \end{array}\right]$$

(b) Convert this system of equations to an augmented matrix $[A|b]$

$$\left\{\begin{array}{r} x_3 - \quad x_2 = 11 - x_1 \\ 5x_1 - 3x_2 + 8x_3 = 13 + x_3 \\ x_1 + \quad x_1 + \quad x_1 = \ 3 \end{array}\right\}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 11 \\ 5 & -3 & 7 & 13 \\ 3 & 0 & 0 & 3 \end{array}\right]$$

(c) Convert this augmented matrix to a vector equation $x_1\vec{\mathbf{v}}_1 + x_2\vec{\mathbf{v}}_2 + x_3\vec{\mathbf{v}}_3 = \vec{\mathbf{b}}$

$$\left[\begin{array}{ccc|c} 5 & 7 & 9 & 11 \\ 2 & 8 & 6 & 4 \\ 3 & 1 & 10 & 12 \end{array}\right]$$

$$X_1\begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} + X_2\begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix} + X_3\begin{bmatrix} 9 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \\ 12 \end{bmatrix}$$

(d) Convert this vector equation to a matrix equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$

$$x_1\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 & 4 \\ 8 & 2 & 5 \\ 9 & 3 & 6 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

(e) Write this stoichiometry problem as a system of equations or augmented matrix (your choice)

$$\underbrace{X_1} P_2I_4 + \underbrace{X_2} P_4 + \underbrace{X_3} H_2O \to \underbrace{X_4} PH_4I + \underbrace{X_5} H_3PO_4$$

$$\begin{array}{c} P \\ I \\ H \\ O \end{array}\left[\begin{array}{ccccc|c} 2 & 4 & 0 & -1 & -1 & 0 \\ 4 & 0 & 0 & -1 & -0 & 0 \\ 0 & 0 & 2 & -4 & -3 & 0 \\ 0 & 0 & 1 & -0 & -4 & 0 \end{array}\right]$$

2. Answer conversion (this page should not involve any serious calculations).

(a) Write down the solution to the system of equations with the following augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\begin{cases} x_1 \; = 7 \\ x_2 \; = 8 \\ x_3 \; = 9 \end{cases}$$

(b) Write down the solution to the system of equations with the following augmented matrix

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & \end{array} \\ \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 3 & 0 & 0 & 5 & 17 \\ 0 & 0 & 1 & 4 & 0 & 0 & 6 & 18 \\ 0 & 0 & 0 & 0 & 1 & 0 & 7 & 29 \\ 0 & 0 & 0 & 0 & 0 & 1 & 8 & 49 \end{array}\right] \end{array}$$

$$\begin{cases} x_1 \; = 17 - 3x_4 - 5x_7, & x_5 \; = 29 - 7x_7 \\ x_2 \; \text{is free}, & x_6 \; = 49 - 8x_7 \\ x_3 \; = 18 - 4x_4 - 6x_7, & x_7 \; \text{is free} \\ x_4 \; \text{is free} & \end{cases}$$

(c) Convert this basic/free variable description to a parametric/vector equation of the solutions $\vec{\mathbf{x}} = \vec{\mathbf{p}} + s\vec{\mathbf{v}}_1 + t\vec{\mathbf{v}}_2 + \ldots$. If there are too many answer blanks, just cross out the extras; if there are too few draw some more.

$$\begin{cases} x_1 & = 10 - 7x_3 \\ x_2 & = 11 + 8x_3 \\ x_3 & \text{is FREE} \\ x_4 & = 5 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -7 \\ 8 \\ 1 \\ 0 \end{bmatrix} + \cancel{\begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}}$$

(d) Convert this basic/free variable description to a parametric/vector equation of the solutions $\vec{\mathbf{x}} = \vec{\mathbf{p}} + s\vec{\mathbf{v}}_1 + t\vec{\mathbf{v}}_2 + \ldots$. If there are too many answer blanks, just cross out the extras; if there are too few draw some more.

$$\begin{cases} x_1 & = 10 - 7x_3 + 1x_5 \\ x_2 & = 11 + 8x_3 - x_5 \\ x_3 & \text{is FREE} \\ x_4 & = 5 \\ x_5 & \text{is FREE} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -7 \\ 8 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(e) Convert this parametric/vector equation into a basic/free description

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 5 \end{bmatrix} + s \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 7 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} x_4$$

$$\begin{cases} x_1 \; = 1 + 6x_2 + 7x_4, & x_4 \; \text{is free} \\ x_2 \; \text{is free}, & x_5 \; = 5 \\ x_3 \; = 3 + 9x_4, & \end{cases}$$

$x_2 = 0 + s(1) + t(0)$

$x_2 = s$, so just call it "x_2"

3. Apply row operations to these matrices until they are in reduced (row) echelon form

(a) Convert the following matrix to RREF. Make sure to show your work clearly. Answers without justification receive no credit.

$$\left[\begin{array}{cc|c} 1 & -3 & -25 \\ -3 & 11 & 95 \end{array}\right] \xrightarrow{R_2+3R_1} \left[\begin{array}{cc|c} 1 & -3 & -25 \\ 0 & 2 & 20 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{cc|c} 1 & -3 & -25 \\ 0 & 1 & 10 \end{array}\right] \xrightarrow{R_1+3R_2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 10 \end{array}\right]$$

$$\begin{array}{lcc|c} R_2 & -3 & 11 & 95 \\ +3R_1 & 3 & -9 & -75 \\ \hline \text{new } R_2 & 0 & 2 & 20 \end{array} \qquad \begin{array}{lcc|c} R_1 & 1 & -3 & -25 \\ +3R_2 & 0 & 3 & 30 \\ \hline \text{new } R_1 & 1 & 0 & 5 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 10 \end{array}\right]$$

(b) Convert the following matrix to RREF. Make sure to show your work clearly. Answers without justification receive no credit.

$$\left[\begin{array}{cccccc|c} 1 & -1 & -3 & 5 & 1 & 6 & 47 \\ 0 & 1 & 5 & 1 & 1 & 15 & 25 \\ 2 & -2 & -6 & 12 & 1 & 3 & 100 \\ 0 & 2 & 10 & 2 & 3 & 39 & 60 \end{array}\right] \xrightarrow[R_4-2R_2]{R_3-2R_1} \left[\begin{array}{cccccc|c} 1 & -1 & -3 & 5 & 1 & 6 & 47 \\ 0 & 1 & 5 & 1 & 1 & 15 & 25 \\ 0 & 0 & 0 & 2 & -1 & -9 & 6 \\ 0 & 0 & 0 & 0 & 1 & 9 & 10 \end{array}\right]$$

$$\xrightarrow[R_3+R_4]{R_1+R_2} \left[\begin{array}{ccccccc} 1 & 0 & 2 & 6 & 2 & 21 & 72 \\ 0 & 1 & 5 & 1 & 1 & 15 & 25 \\ 0 & 0 & 0 & 2 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 1 & 9 & 10 \end{array}\right] \xrightarrow{R_3/2} \left[\begin{array}{ccccccc} 1 & 0 & 2 & 6 & 2 & 21 & 72 \\ 0 & 1 & 5 & 1 & 1 & 15 & 25 \\ 0 & 0 & 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & 9 & 10 \end{array}\right]$$

$$\xrightarrow[R_1-6R_3-2R_4]{R_2-R_3-R_4} \left[\begin{array}{cccccc|c} 1 & 0 & 2 & 0 & 0 & 3 & 4 \\ 0 & 1 & 5 & 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & 9 & 10 \end{array}\right]$$

4. Spans and linear combinations. Calculations on this page are supposed to nice, but show your work.

(a) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ in the plane spanned by (the origin and) the two vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix}$? If so, write it as a linear combination, if not explain why not.

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 5 & 4 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 1 \\ & & \\ & & \end{array}\right] \leftarrow \text{contradiction, no solution.}$$

No. The plane is all $x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 \\ x_2 \\ 5x_2 \end{bmatrix}$ and so the top two #s need to be equal

(b) Write the vector/parametric equation of the line containing the two points: $\begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 6 \\ 11 \end{bmatrix}$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} + t\left(\begin{bmatrix} 3 \\ 4 \\ 6 \\ 11 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix}$$

(c) Is $\begin{bmatrix} 4 \\ 6 \\ 8 \\ 10 \end{bmatrix}$ in the line from part (b)? Explain why or why not. If you run out of time, just explain what sorts of calculations you would need to do.

$$\begin{bmatrix} 4 \\ 6 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} \quad \text{solve for } t$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} \quad \text{no solution, } t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ t \\ 5t \end{bmatrix} \neq \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

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