1.7: Linear Dependence MA322-007 Feb 3 Worksheet

Why does $x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 5 \end{bmatrix} = \vec{\mathbf{b}}$ either have no solutions or infinitely many solutions?

Be prepared to answer to the class.

"No" versus "infinitely many" depends on the numbers in $\vec{\mathbf{b}}$ of course.
How do you tell which $\vec{\mathbf{b}}$ have any solutions?

Be prepared to answer to the class.

Takeaway: If the columns of a matrix A are linearly independent, then there is at most one solution $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$, never infinitely many. The only way to get infinitely many is (the reason above; fill in the official version here).

Weird question: You should have asked: "hrm, what about the rows? do the rows need to be linearly independent? What's up with that?" So... what IS up with that?

1.8: Linear transformations

Usually we thought of $\vec{\mathbf{b}}$ as given, and tried to solve for $\vec{\mathbf{x}}$. We've started to let $\vec{\mathbf{b}}$ vary, and asked if we can even find a $\vec{\mathbf{x}}$. That led us to just try an $\vec{\mathbf{x}}$, to see which $\vec{\mathbf{b}}$ we got. In other words, we can view a matrix like A as a function that that takes $\vec{\mathbf{x}}$ and gives us $\vec{\mathbf{b}} = A\vec{\mathbf{x}}$ as the answer.

This function $A(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ is better than an ordinary function. It is **linear**. $A(\vec{\mathbf{x}} + \vec{\mathbf{y}}) = A(\vec{\mathbf{x}}) + A(\vec{\mathbf{y}})$ and $A(c\vec{\mathbf{x}}) = cA(\vec{\mathbf{x}})$.

Consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$. What does it do to a vector $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

What does A do the vectors $\vec{\mathbf{e}}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

What must it do to $x_1\vec{\mathbf{e}}_1 + x_2\vec{\mathbf{e}}_2$?

Does $A(\vec{\mathbf{x}}) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ have a solution?

Takeaway: $\vec{\mathbf{x}} \xrightarrow{A} \vec{\mathbf{b}}$

Weird question: If $\vec{\mathbf{x}}$ is small, is $\vec{\mathbf{b}}$ small?

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1.7.1 Solve $A(\vec{\mathbf{x}}) = \vec{\mathbf{b}}$ when $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 5 & 4 \\ 3 & 5 & 1 \end{bmatrix}$, $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\vec{\mathbf{b}} = \begin{bmatrix} 14 \\ 8 \\ 2 \end{bmatrix}$ using the observation that $-3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$.

You shouldn't need to row reduce.

1.7.2 In each matrix decide if the columns are linearly independent. If they are not, cross out the **minimum** number of columns to make them linearly independent.

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \\ 6 & 7 & 13 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 5 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

1.9.1 Write down a matrix that sends $\vec{\mathbf{e}}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ to $\vec{\mathbf{b}}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, but also sends $\vec{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ to $\vec{\mathbf{b}}_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\vec{\mathbf{e}}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ to $\vec{\mathbf{b}}_3 = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$.