1.7: Linear Dependence

A **linear dependence relation** amongst vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, ..., $\vec{\mathbf{v}}_n$ is a sequence of numbers c_1, c_2, ..., c_n such that $c_1\vec{\mathbf{v}}_1 + c_2\vec{\mathbf{v}}_2 + \ldots + c_n\vec{\mathbf{v}}_n = \vec{\mathbf{0}}$.

If A is a matrix whose columns are $\vec{\mathbf{v}}_1$, ..., $\vec{\mathbf{v}}_n$, and $\vec{\mathbf{c}}$ is a vector whose entries are c_1, ..., c_n, then $A\vec{\mathbf{c}} = c_1\vec{\mathbf{v}}_1 + \ldots c_n\vec{\mathbf{v}}_n$.

A linear dependence relation $\vec{\mathbf{c}}$ amognst the columns of A is exactly a homogeneous solution $A\vec{\mathbf{c}} = \vec{\mathbf{0}}$.

Linear dependence amongst the columns of A means there are infinitely many solutions to each $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ for which there is at least one solution. Linear independence amongst the columns of A means there is at most one solution.

A linear dependency relation amongst the rows of A requires the same relation to hold in the rows (entries) of $\vec{\mathbf{b}}$ for there to be a solution to $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. Row reduction finds these dependencies when it finds zero rows. The number of rows of A is equal to number of zero rows in the RREF of A plus the number of linearly independent columns. Each linearly independent column creates a new direction for the image $\vec{\mathbf{b}}$, and so each missing direction creates a new requirement on $\vec{\mathbf{b}}$.

1.8: Linear transformations

This function $A(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ is **linear**: $A(\vec{\mathbf{x}} + \vec{\mathbf{y}}) = A(\vec{\mathbf{x}}) + A(\vec{\mathbf{y}})$ and $A(c\vec{\mathbf{x}}) = cA(\vec{\mathbf{x}})$.

Consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$. What does it do to a vector $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

What does A do the vectors $\vec{\mathbf{e}}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

What must it do to $x_1\vec{\mathbf{e}}_1 + x_2\vec{\mathbf{e}}_2$?

Does $A(\vec{\mathbf{x}}) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ have a solution?

Takeaway: $\vec{\mathbf{x}} \xrightarrow{A} \vec{\mathbf{b}}$

MA322-007 Feb 5 quiz Name:________________________

HW1.7 #13 For what values of h are the columns of A linearly independent? Explain why.
$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 9 & h \\ 3 & 6 & 9 \end{bmatrix}$.

1.9.1 Write down a matrix that sends $\vec{\mathbf{e}}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ to $\vec{\mathbf{b}}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, but also sends $\vec{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ to $\vec{\mathbf{b}}_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\vec{\mathbf{e}}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ to $\vec{\mathbf{b}}_3 = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$.

1.9.2 Write down a matrix that rotates vectors by 90° clockwise around the origin. Hint: What does such a matrix do to $\vec{\mathbf{i}} = \vec{\mathbf{e}}_1$ and $\vec{\mathbf{j}} = \vec{\mathbf{e}}_2$.