MA322-007 Feb 24 Exam Name:________________________

Explain your answers, briefly, on each page.
Numbers without justification are worth no credit.

1. Perform the following arithmetic operations or explain why they are not defined.

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + 100 \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + 100 \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 & 9 \\ 10 & 20 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ a & 0 & 1 \end{bmatrix}^{-1}$ (for a an arbitrary real number)

2. Convert between a description of a linear transformation and its matrix.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation, $T(\vec{\mathbf{e}}_1) = \left[\begin{smallmatrix}1\\2\end{smallmatrix}\right]$, $T(\vec{\mathbf{e}}_2) = \left[\begin{smallmatrix}3\\4\end{smallmatrix}\right]$, and $T(\vec{\mathbf{e}}_3) = \left[\begin{smallmatrix}5\\6\end{smallmatrix}\right]$. What is $T(\vec{\mathbf{v}})$ if $\vec{\mathbf{v}} = \vec{\mathbf{e}}_1 + 100\vec{\mathbf{e}}_2$?

(b) Find a matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ for all $\vec{\mathbf{x}} \in \mathbb{R}^3$, where T is from part (a).

(c) If $S : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation defined by $S(\vec{\mathbf{x}}) = B\vec{\mathbf{x}}$ for $B = \begin{bmatrix} 1 & 10 & 100 \\ 2 & 20 & 200 \end{bmatrix}$, what is $S(\vec{\mathbf{e}}_3)$?

(d) If $F : \mathbb{R}^3 \to \mathbb{R}^3$ switches the $\vec{\mathbf{i}}$ and $\vec{\mathbf{k}}$ components (the "x" and "z" components) then find a matrix C so that $F(\vec{\mathbf{v}}) = C\vec{\mathbf{v}}$ for every $\vec{\mathbf{v}}$ in $\mathbb{R}^3$. **Hint:** What is $F(\vec{\mathbf{e}}_1)$?

3. For $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 9 \end{bmatrix}$.

Find a matrix A satisfying the given properties:

(a) $A^2 = I_2$ but $A \neq \pm I_n$ (Challenge: A has no zero entries)

(b) $AB = 0$ but $BA \neq 0$ (Challenge: find all A)

(c) $CA = I_2$ (Challenge: find all A)

(d) $AD = DA$ but $A \neq 0$, $A \neq I_3$ (Challenge: find all A)

4. For each matrix A explain why it is invertible or not.

(a) $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 0 & 2 \\ 2 & 7 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 2 \\ 0 & 0 & 7 \\ 3 & 2 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$