MA322-007 Feb 24 Practice Exam Name:_____

Explain your answers, briefly, on each page.
Numbers without justification are worth no credit.

1. Perform the following arithmetic operations or explain why they are not defined.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 20 \\ 30 & 40 \\ 50 & 60 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + 10 \begin{bmatrix} 7 & 8 & 9 \\ 7 & 8 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \left(3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$

(e) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \left(3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$

(f) $\begin{bmatrix} 1 & a & a \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$ (for a an arbitrary real number)

2. Convert between a description of a linear transformation and its matrix.

(a) If $T : \mathbb{R}^2 \to \mathbb{R}^4$ is a linear transformation satisfying $T(\vec{\mathbf{e}}_1) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ and $T(\vec{\mathbf{e}}_2) = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$ then what is $T(\vec{\mathbf{v}})$ if $\vec{\mathbf{v}} = 10\vec{\mathbf{e}}_1 + \vec{\mathbf{e}}_2$?

(b) Find a matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ for all $\vec{\mathbf{x}} \in \mathbb{R}^2$, where T is from part (a).

(c) If $S : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation defined by $S(\vec{\mathbf{x}}) = B\vec{\mathbf{x}}$ for $B = \begin{bmatrix} 1 & 10 & 100 \\ 2 & 20 & 200 \end{bmatrix}$, what is $S(\vec{\mathbf{e}}_2)$?

(d) If $R : \mathbb{R}^2 \to \mathbb{R}^2$ rotates vectors 45 degrees clockwise, then find a matrix C so that $R(\vec{\mathbf{v}}) = C\vec{\mathbf{v}}$ for every $\vec{\mathbf{v}}$ in $\mathbb{R}^2$. **Hint:** What is $R(\vec{\mathbf{e}}_1)$?

3. For $B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, and $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$.
Find a matrix A satisfying the given properties:

(a) $A^2 = 0$ but $A \neq 0$ (Challenge: A has no zero entries)

(b) $AB = 0$ but $BA \neq 0$ (Challenge: find all A)

(c) $AC = CA + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (Challenge: find all A)

(d) $AD = DA$ but $A \neq 0$, $A \neq I_3$ (Challenge: find all A)

(e) $EA = I_2$ (Challenge: find all A)

4. For each matrix A explain why it is invertible or not.

(a) $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 4 \\ 6 & 7 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & 0 \\ 6 & 7 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 6 & 7 \\ 0 & 0 & 9 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$