MA322-001 Feb 26 quiz Name:_____

Column spaces

1. $\mathbb{R}^2$ is the set of all points (x, y) in the plane with a specified origin $\vec{0} = (0,0)$. The parabola $y = x^2$ is very important in mathematics. Is it a subspace?

Check the subspace test.

(1) Is $\vec{0}$ on the parabola? $\vec{0} = (0,0)$ has $x=0$, $y=0$, $x^2 = 0 = y$ ✓

(2) Addition stays on? $(a, a^2) + (b, b^2) = (a+b, a^2+b^2)$

has $x = a+b$, $y = a^2+b^2$

$x^2 = (a+b)^2 = a^2 + b^2 + 2ab \neq y$ (if $a \neq 0, b \neq 0$)

Specific problem (1,1) and (2,4) are on parabola, but (3,5) is not.

(3) Multiplication stays on? $c(a, a^2) = (ca, ca^2)$

has $x = ca$, $y = ca^2$

$x^2 = (ca)^2 = c^2a^2 \neq y$ (if $a \neq 0, c \neq 0, c \neq 1$)

Specific problem $\vec{v} = (1,1)$ is on, but $2\vec{v} = (2,2)$ is not

2. $\mathbb{R}^3$ is the set of all points (x, y, z) in space with a specified origin $\vec{0} = (0,0,0)$. W is the span of the columns of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 6 \end{bmatrix}$. Is it a subspace? Can you list the vectors in it?

Yes. Span is always a subspace.

Silly List: all $x\begin{bmatrix}1\\0\\4\end{bmatrix} + y\begin{bmatrix}2\\0\\5\end{bmatrix} + z\begin{bmatrix}3\\0\\6\end{bmatrix}$

Silly Check: (1) $\vec{0}$ is $x=0, y=0, z=0$ ✓ but also $x=1, y=-2, z=1$ ✓ (weird list)

(2) if $\vec{v} = a\begin{bmatrix}1\\0\\4\end{bmatrix} + b\begin{bmatrix}2\\0\\5\end{bmatrix} + c\begin{bmatrix}3\\0\\6\end{bmatrix}$

$\vec{w} = d\begin{bmatrix}1\\0\\4\end{bmatrix} + e\begin{bmatrix}2\\0\\5\end{bmatrix} + f\begin{bmatrix}3\\0\\6\end{bmatrix}$

then $\vec{v} + \vec{w}$ has $x = a+d$, $y = b+e$, $z = c+f$ ✓

(3) if $\vec{v} = a\begin{bmatrix}1\\0\\4\end{bmatrix} + b\begin{bmatrix}2\\0\\5\end{bmatrix} + c\begin{bmatrix}3\\0\\6\end{bmatrix}$

$d\vec{v}$ has $x = da$, $y = db$, $z = dc$ ✓

Better List: all $x\begin{bmatrix}1\\0\\0\end{bmatrix} + z\begin{bmatrix}0\\0\\1\end{bmatrix}$, all $\begin{bmatrix}x\\0\\z\end{bmatrix}$, test: $y = 0$

Function spaces

3. $\mathbb{R}^{\mathbb{R}}$ is all functions whose domain is the real numbers. Show that polynomial functions are a subspace. Make sure to clearly state what $\vec{0}$ is.

$\mathbb{R}^{\mathbb{R}}$ is all $f:\mathbb{R}\to\mathbb{R}$ (like $\sin(x)$, $\cos(x)$, e^x, x^2, but not $\sqrt{x}$, $\ln(x)$)

$\vec{0}$ is the zero function that always returns 0, so $0\cdot\sin(x)$, $0\cdot x$, x^2-x^2, etc

① $\vec{0} = x^2 - x^2$ or $\vec{0} = 0\cdot x$ writes $\vec{0}$ as a (weird) polynomial

② $(a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0) + (b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0)$
$= (a_n+b_n)x^n + (a_{n-1}+b_{n-1})x^{n-1} + \cdots + (a_1+b_1)x + (a_0+b_0)$

③ $c(a_n x^n + \cdots + a_0) = (ca_n)x^n + \cdots + (ca_0)$

Easier: Polynomials are linear combinations of $\{1, x, x^2, x^3, \ldots\}$

Spans are always subspaces.

4. The differential equation $y' = y$ specifies a requirement on functions in $\mathbb{R}^{\mathbb{R}}$ (namely, that they be differentiable and that their derivative is equal to themselves). Does this define a subspace? Can you "list" the vectors in it?

From MA113 we know that if $g = f(x)$, then y is differentiable (since $y' = f(x)$ is a function from $\mathbb{R}$ to $\mathbb{R}$, $f'(x)$ is always defined)

~~MVT says~~ if we look at $g(x) = f(x)/e^x$

then $g'(x) = (f'(x)e^x - f(x)e^x)/(e^x)^2 = (f'-f)/e^x = 0$

since $f' = f$. MVT says $g' = 0$ means g is constant.

so $f(x) = g\cdot e^x$, where g is just a number.

This is just Span$\{e^x\}$ so yes a subspace.

Different Check

① $\vec{0}$? well $0' = 0$ so ✓

② if $f' = f$ and $g' = g$, then $(f+g)' = f' + g' = f + g$ ✓

③ if $f' = f$ and c is constant, then $(cf)' = cf' = cf$ ✓