MA322-007 Mar 3 Quiz

Name: Key

HW#4. Is the following a subspace? Show the subspace check.
$W = \{(x,y) : 3x+2y=5\} \subseteq \mathbb{R}^2$

☹ (0) $\vec{0} = (0,0)$ $x=0, y=0$ but $3(0)+2(0)=0 \neq 5$ ☹

☹ (+) $\vec{u}=(1,1)$, $\vec{v}=(3,-2)$ in W but $\vec{u}+\vec{v}=(4,-1)$ has $3(4)+2(-1)=10\neq 5$ ☹

☹ (·) $\vec{u}=(1,1)$ in W, $c=2$ but $c\cdot\vec{u}=(2,2)$ has $3(2)+2(2)=10\neq 5$ ☹

HW#4' Is the following a subspace? Show the subspace check.
$W = \{(x,y) : 3x+2y=0\} \subseteq \mathbb{R}^2$

✓ (0) $\vec{0}=(0,0)$ $x=0, y=0$ and $3(0)+2(0)=0$ ✓ ☺

✓ (+) If $\vec{u}=(a,b)$ $\vec{v}=(c,d)$ in W then $3a+2b=0$, $3c+2d=0$; so $3(a+c)+2(b+d)=0$ so $(a+c,b+d)$ in W

✓ (·) " " in W, $c\in\mathbb{R}$, then $0=c(3a+2b)=3(ac)+2(bc)$ so (ac,bc) in W

HW#5 Is the following a subspace? Show the subspace check.
$W = \{t \mapsto at^2 : a \in \mathbb{R}\} \subseteq \mathbb{R}^{\mathbb{R}}$

✓ (0) $\vec{0} = 0\cdot t$ is also $0\cdot t^2$ so $a=0$ shows $\vec{0}$ in W

✓ (+) If $b\cdot t^2$, $c\cdot t^2$ in W then $b\cdot t^2+c\cdot t^2=(b+c)\cdot t^2$ in W $(a=b+c)$

✓ (·) If $b\cdot t^2$ in W, $c\in\mathbb{R}$ then $c(b\cdot t^2)=(cb)\cdot t^2$ in W $(a=cb)$

HW#6 Is the following a subspace? Show the subspace check.
$W = \{t \mapsto t^2 + a : a \in \mathbb{R}\} \subseteq \mathbb{R}^{\mathbb{R}}$

☹ (0) $\vec{0} = 0\cdot t$ but $t^2+a\neq\vec{0}$ for any a ("$a=-t^2$" is a vector not $a\in\mathbb{R}$ as required)

☹ (+) $(t^2+1)+(t^2+2)=2t^2+3$

↑ oh no!

☹ (·) $2(t^2+1)=2t^2+2$

↑ oh no!

HW#13 Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

(a) How many vectors are in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$? Is $\vec{w} \in \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

3: $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$

No. $\vec{w} \neq \vec{v}_1$
$\vec{w} \neq \vec{v}_2$
$\vec{w} \neq \vec{v}_3$

This is about subsets.
Small and boring by themselves

(b) How many vectors are in Span($\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$)? Is $\vec{w} \in$ Span($\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$)?

∞: $a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3$ for all $a, b, c \in \mathbb{R}$

Yes. $a = 1$
$b = 1$
$c = 0$

Details: $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ -1 & 3 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{array}\right] \xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 5 & 10 & 5 \end{array}\right] \xrightarrow[R_3 - 5R_2]{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$a = 1$, $b = 1 - 2c$, c is free

4.2 Write the null space of $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ as the span of the columns of a matrix N.

Solve $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x = -2y$, $z = 0$ $\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} y$, y free

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} t : t \in \mathbb{R} \right\} = \text{Span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right) = \text{Col}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right)$$