MA322-007 Mar 10 Quiz Name: Key

For each question, explain why your answer works.

4.3: Find five vectors that are in the null space of $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 4 \\ 0 & 1 & 0 & 5 & 6 & 7 \\ 0 & 0 & 1 & 8 & 9 & 0 \end{bmatrix}$

$$Nul(A) = \left\{ \begin{bmatrix} -2 \\ -5 \\ -8 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -3 \\ -6 \\ -9 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_5 + \begin{bmatrix} -4 \\ -7 \\ -0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_6 : x_4, x_5, x_6 \in \mathbb{R} \right\} \qquad x_1 = -2x_4 - 3x_5 - 4x_6$$

so $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ -8 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ -9 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -7 \\ -0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -11 \\ -17 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, etc corresponding to $\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}$ in $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \text{etc} \right\}$

4.3 Which of the following vectors are in the column space of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix} : \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \\ -2 \\ 8 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}?$$

$\vec{a}_1 \; \vec{a}_2 \; \vec{a}_3$

the error

$$\vec{v}_1 = \vec{a}_1 - \vec{a}_2, \quad \vec{v}_2 = \vec{a}_1 - \vec{a}_3, \quad \vec{v}_3 = \vec{a}_1 - \vec{a}_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

so $\vec{v}_3$ is not in $Col(A)$

HW#1. Is $\vec{v} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ in Nul(A) if $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$?

$$A\vec{v} = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = 1 \begin{bmatrix} 3 \\ 6 \\ -8 \end{bmatrix} + 3 \begin{bmatrix} -5 \\ -2 \\ 4 \end{bmatrix} - 4 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 - 15 + 12 \\ 6 - 6 + 0 \\ -8 + 12 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$$

so yes $\vec{v}$ in $Nul(A)$

HW#5. Find a matrix B so that Nul(A) = Col(B) for $A = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & -4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \end{array}$.

$x_1 = 4x_2 - 2x_4$

$x_3 = 5x_4$

$x_5 = 0$

x_2 free

x_4 free

$$Nul(A) = \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ 5 \\ 1 \\ 0 \end{bmatrix} x_4 : x_2, x_4 \in \mathbb{R} \right\}$$

$$= \left\{ \begin{bmatrix} 4 & -2 \\ 1 & 0 \\ 0 & 5 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} : \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} \in \mathbb{R}^2 \right\} = Col(A)$$

HW#7. Is $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c=2 \right\}$ a subspace? No

$\frown$ (0) $\vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $a=0, b=0, c=0$ only possibility but $\overset{a}{0}+\overset{b}{0}+\overset{c}{0} = 0 \neq 2$ $\frown$

$\frown$ (+) fails $= 4$ not 2 $\frown$

$\frown$ ($\cdot$) fails $= 2c$ not 2 $\frown$

Also: if $T(\vec{v}) = [1\ 1\ 1]\vec{v}$ then this is $T^{-1}([2])$ a broken null space.

$Nul([1\ 1\ 1]) = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c=0 \right\}$ zero

$Nul([1\ 1\ 1]) = T^{-1}(\vec{0})$

HW#9. Is $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{array}{rl} a-3b &= 4d, \\ 2a = d+5c \end{array} \right\}$ a subspace? Yes

$\smile$ (0) $\vec{0}$ has $a=0, b=0, c=0, d=0$, $0-3(0)=4(0)$ ✓, $2(0)=0+5(0)$ ✓

(+) $\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ c_1+c_2 \\ d_1+d_2 \end{bmatrix}$ $a_1+a_2-3(b_1+b_2)$ $= a_1-3b_1 + a_2-3b_2$ $= 0 + 0$ $= 0$ $\smile$ etc

Also: This is $Nul\left(\begin{bmatrix} 1 & -3 & 0 & -4 \\ 2 & 0 & -5 & -1 \end{bmatrix} \right)$ so yes

HW#11. Is $\left\{ \begin{bmatrix} x-2y \\ 3+3x \\ 3x+y \\ 2x \end{bmatrix} : x,y \in \mathbb{R} \right\}$ a subspace? No

(0) $\vec{0}$ is broken, $2x=0$ to get bottom, but $3+3x=3$ not 0 in 2nd to top

(+) $(3+3x_1) + (3+3x_2) \neq 3 + 3x_3$ $(= 6 + 3(x_1+x_2))$ no!

($\cdot$) $c(3+3x_1) = 3c + 3(cx_1) \neq 3+3x_3$

Also this is $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix} x + \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ error, not $col\left(\begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 3 & 1 \\ 2 & 0 \end{bmatrix} \right)$

HW#13. Is $\left\{ \begin{bmatrix} x-6y \\ y \\ x \end{bmatrix} : x,y \in \mathbb{R} \right\}$ a subspace? Yes

(0) $\vec{0}$ has $x=0, y=0$

(+) $(x_1, y_1) + (x_2, y_2)$ has $x = x_1 + x_2$, $y = y_1 + y_2$

($\cdot$) $c(x_1, y_1)$ has $x = cx_1$, $y = cy_1$

Also it is just $Col\left(\begin{bmatrix} 1 & -6 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \right)$