1. Are these subspaces? Show the subspace check.

(a) $W = \{(x, y) \in \mathbb{R}^2 : y = (x-1)^2 - x^2\}$

(b) $W = \left\{ \begin{bmatrix} a \\ b \\ 5a + 7b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

(c) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : 5x + 7y - z = 0 \right\}$

(d) $W = \left\{ \begin{bmatrix} x \\ y \\ x^2 + y^2 \end{bmatrix} \in \mathbb{R}^3 : x + y = 1 \right\}$

2. $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, and $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$.

(a) Which of these vectors are in $\mathrm{Nul}(A)$?

(b) Which of these vectors are in $\mathrm{Col}(A)$?

(c) Give lots of examples (5 to infinitely many) of vectors in $\mathrm{Nul}(A)$:

(d) Give lots of examples (5 to infinitely many) of vectors in $\mathrm{Col}(A)$:

3. Convert to nullspaces. Find a matrix B so that $\text{Nul}(B)$ is as required:

(a) $\text{Nul}(B) = \text{Col}(A)$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$.

(b) $\text{Nul}(B) = \left\{ \begin{bmatrix} x \\ y \\ 3x + 7y \\ 8x - 9y \end{bmatrix} : x, y \in \mathbb{R} \right\}$

4. Convert to column spaces. Find a matrix B so that $\mathrm{Col}(B)$ is as required:

(a) $\mathrm{Col}(B) = \mathrm{Nul}(A)$ where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

(b) $\mathrm{Col}(B) = \left\{ \begin{bmatrix} a + 2b \\ 3c - 4d \\ 5a + 6c \\ 7b - 8d \end{bmatrix} : a + b + c + d = 0 \right\}$