MA322-007 Mar 12 Practice Exam Name:_____

1. Are these subspaces? Show the subspace check.

(a) $W = \{(x, y) \in \mathbb{R}^2 : y = (x-1)^2 - 1\}$

(b) $W = \left\{ \begin{bmatrix} a-2b \\ b-2a \\ a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

(c) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^2 : x + y + z = 2x + 3y = 0 \right\}$

(d) $W = \left\{ \begin{bmatrix} a-2b+c \\ b-2a-3c \\ a+b+c \end{bmatrix} : 2a + 3b + 4c = 0 \right\}$

2. $\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$, $\vec{\mathbf{v}}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\vec{\mathbf{v}}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$.

(a) Which of these vectors are in $\mathrm{Nul}(A)$? (Challenge: give the general answer)

(b) Which of these vectors are in $\mathrm{Col}(A)$? (Challenge: give the general answer)

(c) Give lots of examples (5 to infinitely many) of vectors in $\mathrm{Nul}(A)$:

(d) Give lots of examples (5 to infinitely many) of vectors in $\mathrm{Col}(A)$:

3. Convert to nullspaces. Find a matrix B so that $\text{Nul}(B)$ is as required:

(a) $\text{Nul}(B) = \text{Col}(A)$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ -4 & -5 & -6 \end{bmatrix}$.

(b) $\text{Nul}(B) = \left\{ \begin{bmatrix} a+b \\ c-d \\ a+c \\ b-d \end{bmatrix} : a+b+c+d = 0 \right\}$

(c) $\text{Nul}(B) = \left\{ \begin{bmatrix} x \\ y \\ 3x+7y \\ 8x-9y \end{bmatrix} : x, y \in \mathbb{R} \right\}$

4. Convert to column spaces. Find a matrix B so that $\mathrm{Col}(B)$ is as required:

(a) $\mathrm{Col}(B) = \mathrm{Nul}(A)$ where $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{bmatrix}$

(b) $\mathrm{Col}(B) = \left\{ \begin{bmatrix} a+b \\ c-d \\ a+c \\ b-d \end{bmatrix} : a+b+c+d=0 \right\}$

(c) $\mathrm{Col}(B) = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : c = 3a+5b, d = 7a+9b \right\}$