MA322-007 Mar 24 Worksheet - Eigenvalues and eigenvectors

An **eigenpair** of a matrix A is a vector $\vec{v}$ and a number c so that $A\vec{v} = c\vec{v}$. The vector $\vec{v}$ is called the **eigenvector** and the number c is called the **eigenvalue**.

Note that only square matrices can have eigenvectors. Why?

Eigenvectors are very handy: they change matrices into numbers.

Let's consider $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

What is $A\vec{v}$ in terms of $\vec{v}$?

What is $A\vec{w}$ in terms of $\vec{w}$?

What is $A(5\vec{v} + \sqrt{7}\vec{w})$ in terms of $\vec{v}$ and $\vec{w}$?

What is $A(x\vec{v} + y\vec{w})$? Call it $b\vec{v} + c\vec{w}$.

What matrix takes $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ to $\vec{b} = \begin{bmatrix} b \\ c \end{bmatrix}$?

MA322-007 Mar 24 Quiz Name:_____

(5.1a) What are the eigenvalues of $A = \begin{bmatrix} 2 & 0 \\ 0 & \sqrt{7} \end{bmatrix}$?

(5.1b) What are the eigenvectors of $A = \begin{bmatrix} 2 & 0 \\ 0 & \sqrt{7} \end{bmatrix}$?

(5.1c) Give a different matrix with the same eigenvalues:

(5.1d) Give a different matrix with the same eigenvectors:

(5.1e) What are the eigenvalues of $A^3 + 3A + 5I$?