MA322-007 Mar 31 Practice Exam Name:__________________

1. Given a matrix A and a number λ, decide if the number λ is an eigenvalue of A and if so find a corresponding eigenvector $\vec{\mathbf{v}}$.

No (a) $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \end{bmatrix}$, $\lambda = 1$ Check if $A\vec{v} = \lambda\vec{v}$ has nonzero solution

$A - \lambda I \mid \vec{0}$. solve for $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\left[\begin{array}{cc|c} 2-1 & 4 & 0 \\ 3 & -2-1 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 4 & 0 \\ 3 & -3 & 0 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 4 & 0 \\ 0 & -15 & 0 \end{array}\right] \xrightarrow{R_2/-15} \left[\begin{array}{cc|c} 1 & 4 & 0 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1 - 4R_2} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \to \begin{matrix} x=0 \\ y=0 \end{matrix}, \text{ No!}$$

Alt: $1(-3) - 3(4) = -15 \neq 0$ so only the zero $\vec{v} = \vec{0}$ solution. No!

Yes (b) $A = \begin{bmatrix} 8 & 8 \\ -3 & -2 \end{bmatrix}$, $\lambda = 2$

$$\left[\begin{array}{cc|c} 8-2 & 8 & 0 \\ -3 & -2-2 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 6 & 8 & 0 \\ -3 & -4 & 0 \end{array}\right] \xrightarrow{R_2 + \frac{1}{2}R_1} \left[\begin{array}{cc|c} 6 & 8 & 0 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1/6} \left[\begin{array}{cc|c} 1 & 4/3 & 0 \\ 0 & 0 & 0 \end{array}\right] \to \begin{matrix} x = \frac{-4}{3}y \\ y \text{ is free..} \end{matrix}$$

take $y=3$

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

Alt: $6(-4) - (-3)(8) = 0$ so $\vec{v}$ exists... but what is it? :(

Yes (c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$, $\lambda = 3$

$$\left[\begin{array}{ccc|c} 1-3 & 0 & 0 & 0 \\ 0 & 5-3 & -1 & 0 \\ 0 & 6 & 0-3 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 6 & -3 & 0 \end{array}\right] \xrightarrow[R_3 - 3R_2]{R_1/-2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} x=0 \\ y = \frac{1}{2}z \\ z \text{ is free} \end{matrix}$$

take $z=2$

$$\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

Alt: $-2(2)(-3) + 0 + 0 - 0 - (-2)(-1)(6) - 0 = 12 - 12 = 0$
so yes, but $\vec{v} = ?$:(

Double Yes (d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & -3 \\ 0 & 18 & -5 \end{bmatrix}$, $\lambda = 4$

$$\left[\begin{array}{ccc|c} 4-4 & 0 & 0 & 0 \\ 0 & 10-4 & -3 & 0 \\ 0 & 18 & -5-4 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 6 & -3 & 0 \\ 0 & 18 & -9 & 0 \end{array}\right] \xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2/6} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x is free
$y = \frac{1}{2}z$
z is free

$$\vec{v} = x\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

so two directions!

$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ work

but so does $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ etc.

but not $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ (only 2 out of 3 directions)

2. Given a matrix A and a vector $\vec{\mathbf{v}}$, decide if the vector $\vec{\mathbf{v}}$ is an eigenvector of A and if so find the corresponding eigenvalue λ Solve $A\vec{v} = \lambda\vec{v}$ for λ

Yes (a) $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 4 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2(2)+4(-3) \\ 3(2)+(-2)(-3) \end{bmatrix} = \begin{bmatrix} 4-12 \\ 6+6 \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

We need to solve $\begin{cases} 2\lambda = -8 \longrightarrow \lambda = -4 \\ -3\lambda = 12 \longrightarrow \lambda = -4 \end{cases}$ > agree so $\lambda = -4$ is the eigenvalue

No (b) $A = \begin{bmatrix} 8 & 8 \\ -3 & -2 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$ notice same pattern

$$\begin{bmatrix} 8 & 8 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 8(8)+8(3) \\ -3(8)-2(3) \end{bmatrix} = \begin{bmatrix} 64+24 \\ -24-6 \end{bmatrix} = \begin{bmatrix} 88 \\ -30 \end{bmatrix} = \lambda \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

$\begin{cases} 8\lambda = 88 \longrightarrow \lambda = 11 \\ 3\lambda = -30 \longrightarrow \lambda = -10 \end{cases}$ > contradiction, no single # describes A in the $\vec{v}$ direction

Yes (c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix} = 2 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \qquad \lambda = 2$$

No (d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & -3 \\ 0 & 18 & -5 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & -3 \\ 0 & 18 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ -5 \end{bmatrix} \neq \lambda \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$A\vec{v}$ is not in the $\vec{v}$ direction, so A does not act like a single # in the $\vec{v}$ direction

$0\lambda = 0 \longrightarrow \lambda$ is free

$-3\lambda = 0 \longrightarrow \lambda = 0$ > Contradiction

$1\lambda = -5 \longrightarrow \lambda = -5$

3. Given a matrix A, find all of its eigenpairs $(\lambda, \vec{v})$ Solve $\det(A-\lambda I)=0$ for λ

(a) $A = \begin{bmatrix} 4 & -8 \\ -6 & -4 \end{bmatrix}$ $\quad 0 = \det = (\overset{a}{4}-\lambda)(\overset{d}{-4}-\lambda) - (\overset{b}{-8})(\overset{c}{-6})$

$= -16 + \lambda^2 - 48 = \lambda^2 - 64$, so $\lambda = \pm\sqrt{64}$

$\lambda = \pm 8$

$\left(8, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$ $\left[\begin{array}{cc|c} 4-8 & -8 & 0 \\ -6 & -4-8 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -4 & -8 & 0 \\ -6 & -12 & 0 \end{array}\right] \xrightarrow[R_2/-6]{R_1/-4} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 1 & 2 & 0 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = -2y$, y is free $\quad \vec{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\left(-8, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$ $\left[\begin{array}{cc|c} 4+8 & -8 & 0 \\ -6 & -4+8 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 12 & -8 & 0 \\ -6 & 4 & 0 \end{array}\right] \xrightarrow[R_2/-6]{R_1/12} \left[\begin{array}{cc|c} 1 & -\frac{2}{3} & 0 \\ 1 & -\frac{2}{3} & 0 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{cc|c} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = \frac{2}{3}y$, y is free $\quad \vec{v}\ \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 7 \\ -7 & 3 \end{bmatrix}$ $\quad 0 = \det = (3-\lambda)(3-\lambda) - (-7)(7)$

$= 9 - 6\lambda + \lambda^2 + 49 = \lambda^2 - 6\lambda + 58$

$\lambda = \dfrac{6 \pm \sqrt{36-4(58)}}{2} = 3 \pm \dfrac{\sqrt{-196}}{2} = 3 \pm \dfrac{14\sqrt{-1}}{2}$

$\lambda = 3 \pm 7i$ (not a coincidence)

$\left(3+7i, \begin{bmatrix} -i \\ 1 \end{bmatrix}\right)$ $\left[\begin{array}{cc|c} 3-(3+7i) & 7 & 0 \\ -7 & 3-(3+7i) & 0 \end{array}\right] = \left[\begin{array}{cc|c} -7i & 7 & 0 \\ -7 & -7i & 0 \end{array}\right] \xrightarrow[R_2/-7]{R_1/-7i} \left[\begin{array}{cc|c} 1 & i & 0 \\ 1 & i & 0 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = -iy$, y is free

$\left(3-7i, \begin{bmatrix} i \\ 1 \end{bmatrix}\right)$ $\left[\begin{array}{cc|c} 7i & 7 & 0 \\ -7 & 7i & 0 \end{array}\right] \xrightarrow[R_2/-7]{R_1/7i} \left[\begin{array}{cc|c} 1 & -i & 0 \\ 1 & -i & 0 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = iy$, y free $\quad \vec{v} = \begin{bmatrix} i \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\quad 0 = \det = (1-\lambda)(4-\lambda)(6-\lambda) + 2(5)(0) + 3(0)(0) - 0 - 0 - 0$

$= (1-\lambda)(4-\lambda)(6-\lambda)$, so $\lambda = 1$ or $\lambda = 4$ or $\lambda = 6$

$\left(1, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$ $\left[\begin{array}{ccc|c} 0 & 2 & 3 & 0 \\ 0 & 4 & 5 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right] \xrightarrow[R_3/6]{R_2-2R_1} \left[\begin{array}{ccc|c} 0 & 2 & 3 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_2+R_3]{R_1-3R_3} \left[\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1/2} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ x is free, $y = 0$, $z = 0$

$\left(4, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}\right)$ $\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{R_2/5} \begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow[R_3-2R_2]{R_1-3R_2} \begin{bmatrix} -3 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1/-3} \left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$ $x = 2/3\,y$, y is free, $z = 0$ $\quad \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

$\left(6, \begin{bmatrix} 13 \\ 25 \\ 5 \end{bmatrix}\right)$ X $\begin{bmatrix} -5 & 2 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2/-1]{R_1+2R_2} \begin{bmatrix} -5 & 0 & 13 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1/-5} \left[\begin{array}{ccc|c} 1 & 0 & -13/5 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x = 13/5\,z$, $y = 5z$, z is free $\quad \vec{v} = \begin{bmatrix} 13 \\ 25 \\ 5 \end{bmatrix}$

Middle row should have been 0 -2 5, as 4-6 = -2 not -1.
That gives x = (8/5)z, y = (5/2)z,
Final answer v = [16, 25, 10].
Thanks Patrick!

3. Harder [not on exam]

(d) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$

$\det = (1-x)(5-x)(0-x) + 0(-1)(0) + (0)(0)6$
$\quad - 0(5-x)(0) - (1-x)(-1)(6) - 0(0)(0-x)$
$= (1-x)(5-x)(0-x) - (1-x)(-1)(6)$
$= -5x + 6x^2 - x^3 + 6 - 6x$
$= -(x^3 - 6x^2 + 11x - 6)$
$= -(x-1)(x^2 - 5x + 6)$
$= -(x-1)(x-2)(x-3)$

$$\begin{array}{r} x^2 - 5x + 6 \text{ rem } 0 \\ x-1 \,\overline{)\, x^3 - 6x^2 + 11x - 6} \\ \underline{x^3 - x^2 \quad\quad} \\ -5x^2 + 11x \\ \underline{-5x^2 + 5x} \\ 6x - 6 \end{array}$$

so $\lambda = 1$, $\lambda = 2$, $\lambda = 3$

see back for e-vectors

(e) $A = \begin{bmatrix} 2 & 7 & -3 \\ 0 & 11 & -5 \\ 0 & 25 & -11 \end{bmatrix}$

$\lambda = 2$

$(11-\lambda)(-11-\lambda) + 5(25)$
$-121 + \lambda^2 + 125$
$= \lambda^2 + 4$, so $\lambda = \pm 2i$

see back for e-vecs

(f) $A = \begin{bmatrix} 15 & 25 \\ -9 & -15 \end{bmatrix}$

$0 = (15-\lambda)(-15-\lambda) - (25)(-9)$
$= -225 + \lambda^2 + 225$
$= \lambda^2$, $\lambda = \pm\sqrt{0}$, but $A \neq 0$!

$\left[\begin{array}{cc|c} 15-0 & 25 & 0 \\ -9 & -15-0 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 15 & 25 & 0 \\ -9 & -15 & 0 \end{array}\right] \xrightarrow[R_2/-3]{R_1/5} \left[\begin{array}{cc|c} 3 & 5 & 0 \\ 3 & 5 & 0 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 3 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = \frac{-5}{3}y$, y is free

$\vec{v} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ is the only direction where A acts like $\lambda = 0$

in every other direction A acts like $\lambda =$ "dx" infintesimal

$\boxed{\text{HW: Calculate } A^2}$

4. Applications

(a) Find $(A^3 + 5A)$ if $\begin{bmatrix} 7 & 0 \\ 0 & 11 \end{bmatrix}$

$$\begin{bmatrix} 7 & 0 \\ 0 & 11 \end{bmatrix}^3 + 5\begin{bmatrix} 7 & 0 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 7^3 & 0 \\ 0 & 11^3 \end{bmatrix} + \begin{bmatrix} 5(7) & 0 \\ 0 & 5(11) \end{bmatrix}$$

$$= \begin{bmatrix} 7^3 + 5(7) & 0 \\ 0 & 11^3 + 5(11) \end{bmatrix}$$

(b) Solve $(A^3 + 5A)\vec{\mathbf{x}} = 7\vec{\mathbf{v}} + 11\vec{\mathbf{w}}$ if A has eigenpairs $(2, \vec{\mathbf{v}})$ and $(9, \vec{\mathbf{w}})$

$$A\vec{v} = 2\vec{v}, \quad (A^3 + 5A)\vec{v} = (2^3 + 5(2))\vec{v}$$

$$(A^3 + 5A)s\vec{v} = (2^3 + 5(2))s \cdot \vec{v}$$

Set $(2^3 + 5(2)) \cdot s = 7$, $s = 7/(2^3 + 5(2))$

$$\vec{X} = \frac{7}{2^3 + 5(2)}\vec{v} + \frac{11}{9^3 + 5(9)}\vec{\omega}$$

(c) Find $(A^{99} + 5A)(\vec{\mathbf{v}} + \vec{\mathbf{w}})$ if A has eigenpairs $(0.1, \vec{\mathbf{v}})$ and $(1, \vec{\mathbf{w}})$

$$= A^{99}\vec{v} + 5A\vec{v} + A^{99}\vec{w} + 5A\vec{\omega}$$

$$= (0.1)^{99}\vec{v} + 5(0.1)\vec{v} + 1^{99}\vec{\omega} + 5(1)\vec{\omega}$$

$$\approx 0\vec{v} + 0.5\vec{v} + \vec{\omega} + 5\vec{\omega}$$

$$= \frac{1}{2}\vec{v} + 6\vec{\omega}$$

(plus a <u>tiny</u> bit of $\vec{v}$)

3d e-vector

$\lambda = 1$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 6 & 0 \end{bmatrix} \xrightarrow{R_2 - \frac{4}{6}R_3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 6 & 0 \end{bmatrix} \rightarrow$ x is free, $y = 0$, $z = 0$

$\left(1, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$

$\lambda = 2$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & 6 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $x = 0$, $y = \frac{1}{3}z$, z is free

$\left(2, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}\right)$

$\lambda = 3$ $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 6 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $x = 0$, $y = \frac{1}{2}z$, z is free

$\left(3, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}\right)$

3e Evectors

Yuck!

$$\lambda=2 \quad \begin{bmatrix} 0 & 7 & -3 \\ 0 & 9 & -5 \\ 0 & 25 & -9 \end{bmatrix} \xrightarrow{R_3-2R_2-R_1} \begin{bmatrix} 0 & 7 & -3 \\ 0 & 9 & -5 \\ 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_3/4} \begin{bmatrix} 0 & 7 & -3 \\ 0 & 9 & -5 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2+5R_3]{R_1+3R_3}$$

$$\begin{bmatrix} 0 & 7 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2-9R_1\ (/7)]{R_1/7} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{matrix} x \text{ is free} \\ y=0 \\ z=0 \end{matrix} \qquad \vec{V} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(2, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

notice how the top row didn't matter
this is a type of separation of variables
the zeros in the first column do
the separating.

$$\lambda=2i \quad \begin{bmatrix} 2-2i & 7 & -3 \\ 0 & 11-2i & -5 \\ 0 & 25 & -11-2i \end{bmatrix} \xrightarrow[R_3/25]{R_2/11-2i} \begin{bmatrix} 2-2i & 7 & -3 \\ 0 & 1 & \frac{-11-2i}{25} \\ 0 & 1 & \frac{-11-2i}{25} \end{bmatrix} \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|c} 2-2i & 7 & -3 & 0 \\ 0 & 1 & \frac{-11-2i}{25} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{1}{11-2i} = \frac{11+2i}{(11-2i)(11+2i)} = \frac{11+2i}{121+4} = \frac{11+2i}{125}$$

$$\xrightarrow{R_1-7R_2} \left[\begin{array}{ccc|c} 2-2i & 0 & \frac{2+14i}{25} & 0 \\ 0 & 1 & \frac{-11-2i}{25} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{aligned} x &= -\frac{2+14i}{25(2-2i)} z = \frac{3-4i}{25} z \\ y &= \frac{11+2i}{25} z \\ z &\text{ is free} \end{aligned}$$

$$\left(2i, \begin{bmatrix} 3-4i \\ 11+2i \\ 25 \end{bmatrix}\right)$$

$$\text{Check} \quad \begin{bmatrix} 2 & 7 & -3 \\ 0 & 11 & -5 \\ 0 & 25 & -11 \end{bmatrix} \begin{bmatrix} 3-4i \\ 11+2i \\ 25 \end{bmatrix} = \begin{bmatrix} 8+6i \\ -4+22i \\ 50i \end{bmatrix}$$

$$\text{vs} \quad 2i \begin{bmatrix} 3-4i \\ 11+2i \\ 25 \end{bmatrix} = \begin{bmatrix} 6i-8(i)^2 \\ 22i+4(i)^2 \\ 50i \end{bmatrix} = \checkmark$$

3f $A = \begin{bmatrix} 15 & 25 \\ -9 & -15 \end{bmatrix}$ acts like $\lambda = 0$ in the $\vec{v} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ direction

$$A^2 = \begin{bmatrix} 15 & 25 \\ -9 & -15 \end{bmatrix} \begin{bmatrix} 15 & 25 \\ -9 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 15(15) + 25(-9) & 15(25) + 25(-15) \\ -9(15) + (-15)(-9) & -9(25) + -15(-15) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

acts like $\lambda = 0$ in <u>all</u> directions

so $A =$ "$\sqrt{0}$" in a new way