MA322-007 Mar 31 Practice Exam Name:________________________

1. Given a matrix A and a number λ, decide if the number λ is an eigenvalue of A and if so find a corresponding eigenvector $\vec{\mathbf{v}}$.

(a) $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \end{bmatrix}$, $\lambda = 1$

(b) $A = \begin{bmatrix} 8 & 8 \\ -3 & -2 \end{bmatrix}$, $\lambda = 2$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$, $\lambda = 3$

(d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & -3 \\ 0 & 18 & -5 \end{bmatrix}$, $\lambda = 4$

2. Given a matrix A and a vector $\vec{\mathbf{v}}$, decide if the vector $\vec{\mathbf{v}}$ is an eigenvector of A and if so find the corresponding eigenvalue λ

(a) $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 8 & 8 \\ -3 & -2 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

(d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 10 & -3 \\ 0 & 18 & -5 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3. Given a matrix A, find all of its eigenpairs $(\lambda, \vec{\mathbf{v}})$

(a) $A = \begin{bmatrix} 4 & -8 \\ -6 & -4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 7 \\ -7 & 3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

3. Harder [not on exam]

(d) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 6 & 0 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2 & 7 & -3 \\ 0 & 11 & -5 \\ 0 & 25 & -11 \end{bmatrix}$

(f) $A = \begin{bmatrix} 15 & 25 \\ -9 & -15 \end{bmatrix}$

4. Applications

(a) Find $(A^3 + 5A)$ if $\begin{bmatrix} 7 & 0 \\ 0 & 11 \end{bmatrix}$

(b) Solve $(A^3 + 5A)\vec{\mathbf{x}} = 7\vec{\mathbf{v}} + 11\vec{\mathbf{w}}$ if A has eigenpairs $(2, \vec{\mathbf{v}})$ and $(9, \vec{\mathbf{w}})$

(c) Find $(A^{99} + 5A)(\vec{\mathbf{v}} + \vec{\mathbf{w}})$ if A has eigenpairs $(0.1, \vec{\mathbf{v}})$ and $(1, \vec{\mathbf{w}})$