MA322-007 Apr 1 Exam Name:_____

1. Given a matrix A and a number λ, decide if the number λ is an eigenvalue of A and if so find a corresponding eigenvector $\vec{\mathbf{v}}$.

(a) $A = \begin{bmatrix} -3 & 5 \\ -4 & 6 \end{bmatrix}$, $\lambda = 1$

(b) $A = \begin{bmatrix} -6 & 10 \\ -2 & 6 \end{bmatrix}$, $\lambda = 2$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 4 & -6 \\ -1 & 1 & -1 \end{bmatrix}$, $\lambda = 3$

(d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 4 & -4 & 8 \\ 2 & -4 & 8 \end{bmatrix}$, $\lambda = 4$

2. Given a matrix A and a vector $\vec{\mathbf{v}}$, decide if the vector $\vec{\mathbf{v}}$ is an eigenvector of A and if so find the corresponding eigenvalue λ

(a) $A = \begin{bmatrix} -3 & 5 \\ -4 & 6 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

(b) $A = \begin{bmatrix} -6 & 10 \\ -2 & 6 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 4 & -6 \\ -1 & 1 & -1 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 4 & 0 & 0 \\ 4 & -4 & 8 \\ 2 & -4 & 8 \end{bmatrix}$, $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

3. Given a matrix A, find all of its eigenpairs $(\lambda, \vec{\mathbf{v}})$

(a) $A = \begin{bmatrix} -7 & 20 \\ -2 & 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 & 0 \\ 4 & -6 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

4. Applications

(a) Find $(A^5 + 3A)$ if $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

(b) Find $(A^6 + 5A)(3\vec{\mathbf{v}} + \vec{\mathbf{w}})$ if A has eigenpairs $(0.1, \vec{\mathbf{v}})$ and $(2, \vec{\mathbf{w}})$

(c) Solve $(A^7 + 2A)\vec{\mathbf{x}} = 3\vec{\mathbf{v}} + 5\vec{\mathbf{w}}$ if A is a three by three matrix with eigenpairs $(-1, \vec{\mathbf{u}})$, $(1, \vec{\mathbf{v}})$ and $(9, \vec{\mathbf{w}})$