MA322-007 Apr 21 - Exam Name: ____________________

1. $\vec{\mathbf{a}} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\vec{\mathbf{b}} = \begin{bmatrix} 7 \\ 5 \\ -5 \\ -1 \end{bmatrix}$, $\vec{\mathbf{c}} = \begin{bmatrix} 14 \\ 10 \\ -10 \\ -2 \end{bmatrix}$.

(a) Compute $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

(b) Compute $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$

(c) Compute $\|\vec{\mathbf{b}}\|$

(d) Compute $\|\vec{\mathbf{c}}\|$

(e) Compute $\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{\vec{\mathbf{b}} \cdot \vec{\mathbf{b}}}\vec{\mathbf{b}}$

(f) Compute $\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}}{\vec{\mathbf{c}} \cdot \vec{\mathbf{c}}}\vec{\mathbf{c}}$

(g) Explain what happened for (e) versus (f)

2. $G = \begin{bmatrix} \uparrow & \uparrow & \uparrow \\ \vec{\mathbf{g}}_1 & \vec{\mathbf{g}}_2 & \vec{\mathbf{g}}_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.7 \\ 0.5 & 0.5 & -0.7 \\ 0.5 & -0.5 & 0.1 \\ 0.5 & -0.5 & -0.1 \end{bmatrix}$ and $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 100 \\ 100 \\ 100 \end{bmatrix}$ and $G^T G = I_3$.

(a) Find $\vec{\mathbf{y}}$ so that $G\vec{\mathbf{y}}$ is as close as possible to $\vec{\mathbf{v}}$.

(b) How close is that?

3. $A = \begin{bmatrix} \uparrow & \uparrow & \uparrow & \uparrow \\ \vec{\mathbf{a}}_1 & \vec{\mathbf{a}}_2 & \vec{\mathbf{a}}_3 & \vec{\mathbf{a}}_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 1 & 1 & 7 & 1 \\ 1 & 1 & -7 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Find orthonormal vectors $\vec{\mathbf{g}}_1, \vec{\mathbf{g}}_2, \vec{\mathbf{g}}_3, \vec{\mathbf{g}}_4$ that span $\text{Col}(A)$. In other words, $\text{span}(\vec{\mathbf{g}}_1, \vec{\mathbf{g}}_2, \vec{\mathbf{g}}_3, \vec{\mathbf{g}}_4) = \text{span}(\vec{\mathbf{a}}_1, \vec{\mathbf{a}}_2, \vec{\mathbf{a}}_3, \vec{\mathbf{a}}_4)$, $\vec{\mathbf{g}}_i \cdot \vec{\mathbf{g}}_i = 1$, and $\vec{\mathbf{g}}_i \cdot \vec{\mathbf{g}}_j = 0$ if $i \neq j$.

4. $A = \begin{bmatrix} \uparrow & \uparrow & \uparrow \\ \vec{\mathbf{a}}_1 & \vec{\mathbf{a}}_2 & \vec{\mathbf{a}}_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 7 \\ 1 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 0 \\ 100 \\ 100 \\ 100 \end{bmatrix}$

(a) Use RREF to show that there is no $\vec{\mathbf{x}}$ with $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

(b) Use G to find the $\vec{\mathbf{x}}$ with $A\vec{\mathbf{x}}$ as close $\vec{\mathbf{b}}$ as possible.

$G = \begin{bmatrix} \uparrow & \uparrow & \uparrow \\ \vec{\mathbf{g}}_1 & \vec{\mathbf{g}}_2 & \vec{\mathbf{g}}_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.7 \\ 0.5 & 0.5 & -0.7 \\ 0.5 & -0.5 & 0.1 \\ 0.5 & -0.5 & -0.1 \end{bmatrix}$ Note that $G^T G = I_3$ and $G^T A = \begin{bmatrix} 2 & 1 & 7 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix}$