#### MA111: Contemporary mathematics

Jack Schmidt

No entrance or exit quiz today (pick-up your mini-exam).

Schedule:

- HW 1 part 2 is due 11:59pm tonight.
- HW 2 is due 11:59am Thursday, Sep 17th, 2015
- HW 3 is due 7am Tuesday, Sep 22nd, 2015
- Exam 1 is in-class on Thursday, Sep 24th, 2015

Today we cover elimination methods and two fairness criteria

- Who wins plurality? Who loses the worst?
- Who wins each head-to-head match up?

6 4 3 2 С С Α В 1st В С В В 2nd С А А D 3rd D D D Α 4th

• So who wins pairwise-comparison? Who loses the worst?

• What if we got rid of D entirely?

### Head-to-head and elimination

- Who wins plurality? Who loses the worst?
- Who wins each head-to-head match up?

	6	4	3	2
1st	Α	В	С	С
2nd	В	С	В	В
3rd	С	Α	Α	D
4th	D	D	D	Α

A vs B	A vs C	A vs D	B vs C	B vs D	C vs D
6 to 4+3+2	6 to 4+3+2	6+4+3 to 2	6+4 to 3+2	6+4+3+2 to 0	6+4+3+2 to 0
6 to 9	6 to 9	13 to 2	10 to 5	15 to 0	15 to 0
B wins	C wins	A wins	B wins	B wins	C wins

So who wins pairwise-comparison? Who loses the worst?

A: 1, B: 3, C: 2, D: 0. B wins everything! (D loses everything!)

• What if we got rid of D entirely?

Then A would lose everything! B would have the least first-place votes.

- An **elimination method** removes the worst candidate until only the best is left.
- Pairwise-comparison with elimination removes the candidate that loses the most head-to-heads (and repeat)
- Plurality with elimination removes the candidate that has the least first place votes (and repeat)
- Survivor style elimination removes the candidate with the most last place votes (and repeat)
- Let's try each method on this one

- A preference schedule writes down rankings of candidates
- A **voting method** takes a preference schedule and declares one candidate the winner
- A fairness criterion takes a voting method and says whether it is good

- **Majority criterion**: if a candidate has more than half of the first places votes, the voting method should declare them the winner
- **Condorcet criterion**: if a candidate wins every head-to-head, then the voting method should declare them the winner



A wins plurality; B wins survivor, pairwise comparison, and pairwise comparison with elimination; C wins plurality with elimination

- Is there a majority candidate?
- Is there a condorcet candidate?
- What does this say about each voting method and the majority fairness criterion?
- What does this say about each voting method and the condorcet fairness criterion?

### Inconclusive

- Majority criterion is **inconclusive** in this case. There is no majority candidate, so the fairness criterion doesn't require anything.
  Everybody automatically "passes this time", but they could fail in the future; we don't know.
- Condorcet criterion is **violated** by plurality and plurality with elimination, since the methods were required to make B (the Condorcet candidate) win, but they didn't.
- Condorcet criterion is **inconclusive** in this case for survivor, pairwise comparison, and pairwise comparison with elimination, since the methods were required to make B win and they did. They "passed this time" but they could fail in the future; we don't know.

# Proofs - Can you explain why something always works?

- A major idea in mathematics is tring to decide "always"
- Can you explain why pairwise comparison method always passes the Condorcet criterion?

Pairwise comparison with elimination too?

• Can you explain why plurality method always passes the Majority criterion?

Plurality with elimination too?

• What about survivor?

# Proofs - Can you explain why something always works?

- A major idea in mathematics is tring to decide "always"
- Can you explain why pairwise comparison method always passes the Condorcet criterion?

Pairwise comparison with elimination too?

• Can you explain why plurality method always passes the Majority criterion?

Plurality with elimination too?

• What about survivor?

Survivor fails both in this very similar case

