# MA111: Contemporary mathematics

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Schedule:

- HW 3 is due 11:59pm tonight, Sep 22nd, 2015
- Exam 1 is in-class on Thursday, Sep 24th, 2015

Today we review the fairness criteria and cover Arrow's theorem

# Fairness criteria

- A **voting method** takes a **preference schedule** and determines the **winning candidate**
- Some voting methods are awful: "The winning candidate is the candidate who receives the most 2nd to last place votes."
- A **fairness criterion** is a specific requirement for a voting method not to be **broken**
- Some are pretty clear: "If absolutely every voter ranks the same candidate as their first place choice, then the voting method should choose that candidate as the winner."

| | 5 | 3 | 2 |
|---|---|---|---|
| 1st | A | A | A |
| 2nd | B | C | D |
| 3rd | C | D | B |
| 4th | D | B | C |

# Our main fairness criteria

- "Strong candidates should win"

  **Majority fairness criterion** - if one candidate receives more than half of the first place votes, then the voting method must declare that candidate the winner

  **Condorcet fairness criterion** - if one candidate beats each other candidate in head-to-head matchups, then the voting method must declare that candidate the winner

- "Weak candidates should not change the winner"

  **IIA** - if a candidate is not declared the winner in one election, then the voting method should declare the same winner whether or not that losing candidate is eliminated or not

- "Votes are good"

  **Monotonicity** - a voting method should declare the same winner even if some voters move the winner up on their ballots while leaving the other candidates in the same relative order

# How do our methods stack up

- Majority:

  Plurality, Plurality with Elimination, Pairwise-Comparison (and most head to heads) pass

  Borda Count (and most point systems) fails

- Condorcet:

  Pairwise-Comparison (and most head to heads) pass

  Plurality and Borda Count (and most point systems), Plurality with Elimination (and most elimination methods) fail

- IIA: Most systems fail ("approval voting" passes)

- Monotonicity:

  Plurality and Borda Count (and most point systems) and Pairwise-Comparison (and most head-to-head methods) pass

  Plurality with elimination (and most elimination methods) fail

# Impossibility theorems

- We studied other systems, maybe there is a better method?
- **Arrow's:** Every possible voting method violates one of Majority, IIA, or Monotonicty.
- **Gibbard-Satterthwaite's:** The only voting methods in which voting honestly is always an optimal startegy for each voter is (a) dictatorship by a voter or (b) there are candidates that can never win, no matter how the voters vote.