

Vowels: (mod 5)
Consonants: (mod 21)

Vowel(and y)s: (mod 6)
Consonant(not y)s: (mod 20)

1. For each number, list 4 equivalent numbers of the same type

$$1 \pmod{5}: 1, 6, 11, 16, 21, 26, 31, 36, \dots, 101, 106, \dots \quad -4, -9, -14, -19, \dots$$

$$1 \pmod{6}: 1, 7, 13, 19, 25, 31, 37, 43, \dots, 601, 607, \dots \quad -5, -11, -17, -23, \dots$$

$$1 \pmod{21}: 22, 43, 64, 85, 106, \dots, -20, \dots$$

$$1 \pmod{20}: 21, 41, 61, 81, 101, \dots, 201, 221, \dots, -19, -39, \dots$$

$$40 \pmod{5}: 45, 540, 5040, 56040, \dots, -10, -15, \dots$$

$$40 \pmod{6}: 46, 640, 6040, 60040, \dots, -20, -26, \dots$$

$$40 \pmod{21}: 61, 250, 2140, 21040, \dots, -170, -191, \dots$$

$$40 \pmod{20}: 60, 240, 2040, 20040, \dots, -160, -180, \dots$$

2. For any number $x \pmod{n}$, there is a unique number y between 1 and n with $x \equiv y \pmod{n}$. This number is called the **standard representative** of the number. Find this number y for each x :

$$0 \pmod{5}: 5$$

$$0 \pmod{6}: 6$$

$$0 \pmod{21}: 21$$

$$0 \pmod{20}: 20$$

$$-1 \pmod{5}: 4$$

$$-1 \pmod{6}: 5$$

$$-1 \pmod{21}: 20$$

$$-1 \pmod{20}: 21$$

$$40 \pmod{5}: 5$$

$$40 \pmod{6}: 4$$

$$40 \pmod{21}: 19$$

$$40 \pmod{20}: 20$$

3. For each arithmetic problem, write down the standard representative of the answer.

$$2 + 3 \pmod{5}: 5$$

$$2 + 3 \pmod{6}: 5$$

$$2 + 30 \pmod{5}: 2$$

$$2 + 30 \pmod{6}: 2$$

$$2 \times 30 \pmod{5}: 0$$

$$2 \times 30 \pmod{6}: 0$$

$$11 + 22 \pmod{5}: 3$$

$$11 + 22 \pmod{6}: 3$$

$$11 \times 22 \pmod{5}: 2$$

$$11 \times 22 \pmod{6}: 2$$

4. For each number, list ALL standard representatives of numbers that work:

- (a) Which numbers double to 2 (mod 5):
 Before 1 2 3 4 5
 After 2 4 1 3 5
 Only 1
- (b) Which numbers double to 2 (mod 6):
 Before 1 2 3 4 5 6
 After 2 4 6 2 4 6
 Both 1 and 4
- (c) Which numbers double to 1 (mod 5):
 Only 3
- (d) Which numbers double to 1 (mod 6):
 None

(e) How many answers can there be?

Mod 5: Always 1, Mod 6: 0 or 2

(f) How do you tell how many answers there are?

Mod 5: Always 1, Mod 6: $\begin{cases} 0 & \text{if odd} \\ 2 & \text{if even} \end{cases}$

(g) How do consonants (mod 21) and consonant(y)s (mod 20) work?

Mod 21: Like Mod 5, Mod 20: Like Mod 6

5. For each number, list ALL standard representatives of numbers that work:

- (a) Triple to 3 (mod 5):
 Before 1 2 3 4 5
 After 3 1 4 2 5
 Only 3
- (b) Triple to 3 (mod 6):
 Before 1 2 3 4 5 6
 After 3 6 3 6 3 6
 All of 1, 3, 5
- (c) Triple to 1 (mod 5):
 Only 2
- (d) Triple to 1 (mod 6):
 None
- (e) Triple to 2 (mod 5):
 None
- (f) Triple to 2 (mod 6):
 None

(g) How many answers can there be?

Mod 5: Always One, Mod 6: Three or None

(h) How do you tell how many answers there are?

Mod 5: Always One, Mod 6: $\begin{cases} 3 & \text{if divisible by 3} \\ 0 & \text{otherwise} \end{cases}$

(i) How do consonants (mod 21) and consonant(y)s (mod 20) work?

Mod 21: Like Mod 6, Mod 20: Like Mod 5

6. Instead of triple, maybe it is quadruple, quintuple, or even multiply by 23.

(a) How many answers can there be?

Quad: Always one, None or Two, None or Two or 4

(b) How do you tell how many answers there are?

Quad: Is the number you want to divide divisible by 4?