Easy way: Reduce the question first, then reduce your answer.

1. Give standard representations for each arithmetic problem (preferably without a calculator):

$2+3 \pmod{21}$

5

$2 \times 3 \pmod{21}$

6

$20+30 \pmod{21}$

$\frac{-21}{-1} \quad \frac{-21}{9} = 8$

$-1 + 9$

8

$20 \times 30 \pmod{21}$

$\frac{-21}{-1} \times \frac{-21}{9} = -9^{+21} = 12$ ★

$6 \times 3 \pmod{5}$

$\frac{-5}{1} \times 3$ 3

$22 \times 17 \pmod{21}$

$\frac{-21}{1} \times 17$ 17

$181+212 \pmod{10}$

$\frac{-180}{1} \quad \frac{-210}{2}$

$1 + 2$

3

$181+212 \pmod{100}$

$\frac{-100}{81} \quad \frac{-200}{12}$

$81 + 12$

93

$181 \times 212 \pmod{10}$

$\frac{-180}{1} \quad \frac{-210}{2}$

1×2

2

$181 \times 212 \pmod{20}$

$\frac{-160}{1} \quad \frac{-200}{12}$

1×12

12

2. Fill in the third multiplication table, and for each row decide if the number is a unit, a zero divisor, or a zero. The first two are done for you.

mod 5	1	2	3	4	5	Type
1	1	2	3	4	5	Unit
2	2	4	1	3	5	Unit
3	3	1	4	2	5	Unit
4	4	3	2	1	5	Unit
5	5	5	5	5	5	Zero

mod 6	1	2	3	4	5	6	Type
1	1	2	3	4	5	6	Unit
2	2	4	6	2	4	6	Zero divisor
3	3	6	3	6	3	6	Zero divisor
4	4	2	6	4	2	6	Zero divisor
5	5	4	3	2	1	6	Unit
6	6	6	6	6	6	6	Zero

mod 10	1	2	3	4	5	6	7	8	9	10	Type
1	(1)	2	3	4	5	6	7	8	9	10	Unit
2	2	4	6	8	(10)	2	4	6	8	10	Zero Divisor
3	3	6	9	2	5	8	(1)	4	7	10	Unit
4	4	8	2	6	(10)	4	8	2	6	10	Zero Divisor
5	5	10	5	(10)	5	10	5	10	5	10	Zero divisor
6	6	2	8	4	(10)	6	2	8	4	10	Zero Divisor
7	7	4	(1)	8	5	2	9	6	3	10	Unit
8	8	6	4	2	(10)	8	6	4	2	10	Zero divisor
9	9	8	7	6	5	4	3	2	(1)	10	Unit
10	10	10	10	10	10	10	10	10	10	10	Zero

Unit: has (1)
ZD: has (10) early
Zero: (10) all the way

3. Here is the start of the multiplication table:

mod 20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Type
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Unit
2	2	4	6	8	10	12	14	16	18	20	2	4	6	8	10	12	14	16	18	20	Zero Divisor
3	3	6	9	12	15	18	1	4	7	10	13	16	19	2	5	8	11	14	17	20	Unit
4	4	8	12	16	20	4	8	12	16	20	4	8	12	16	20	4	8	12	16	20	Zero Divisor
5	5	10	15	20	5	10	15	20	5	10	15	20	5	10	15	20	5	10	15	20	Zero divisor

(a) List 3 different units mod 20.

1, 3 and .... not 6 ($6 \times 10 = 20 \bmod 20$) 7 ($7 \times 3 = 1 \bmod 20$)

(b) How do you tell if a number is a zero divisor / zero?

The 20 shows up early

(c) How do you tell if a number is a unit?

The 1 shows up at all

(d) Notice that $3 \times 7 \equiv 1 \pmod{20}$. Find a number that goes to 5 (mod 20) after being multiplied by 7.

$15 \times 7 = (5 \times 3) \times 7 = 5 \times (3 \times 7) = 5 \times 1 = 5$ 15

(e) Explain how to divide by 7 (mod 20)

Multiply by 3, Example: $\frac{8}{7} = 8 \times 3 = 24 = 4$ $7 \times 4 = 28$ -20 8

4. Here is a way to find the pairs of units. Let's try mod 21. Pairs of units are numbers that multiply to 1. So we need to factor 1. Regular 1 does not factor. $1 + 21 = 22$ does factor. $2 \times 11 = 22 \equiv 1 \pmod{21}$ so 2 and 11 are pairs of units.

Sometimes we don't get anything new. We keep going and try to factor $1 + 21 + 21 = 43$, but 43 doesn't factor ($1 \times 43 = 43$ but $43 \equiv 1 \pmod{21}$ so we are just saying $1 \times 1 \equiv 1$).

(a) Keep going to find a pair of units with 4 and another pair with 8: $43 + 21 = 64$

$4 \times 16 = 64 \equiv 1 \bmod 21$

$8 \times 8 = 64 \equiv 1 \bmod 21$

(b) Keep going to find a pair of units with 5: $64 + 21 = 85$

$5 \times 17 = 85 \equiv 1 \bmod 21$

(c) Can you find one for 10? $85 + 21 = 106$, $+21 = 127$, $+21 = 148$, $+21 = 169$, $+21 = 190$

$10 \times 19 = 190 \equiv 1 \bmod 21$