1. Give standard representations for each arithmetic problem (preferably without a calculator):

$2 + 3 \pmod{21}$ $2 \times 3 \pmod{21}$

$20 + 30 \pmod{21}$ $20 \times 30 \pmod{21}$

$6 \times 3 \pmod{5}$ $22 \times 17 \pmod{21}$

$181 + 212 \pmod{10}$ $181 + 212 \pmod{100}$

$181 \times 212 \pmod{10}$ $181 \times 212 \pmod{20}$

2. Fill in the third multiplication table, and for each row decide if the number is a unit, a zero divisor, or a zero. The first two are done for you.

mod 5	1	2	3	4	5	Type
1	1	2	3	4	5	Unit
2	2	4	1	3	5	Unit
3	3	1	4	2	5	Unit
4	4	3	2	1	5	Unit
5	5	5	5	5	5	Zero

mod 6	1	2	3	4	5	6	Type
1	1	2	3	4	5	6	Unit
2	2	4	6	2	4	6	Zero divisor
3	3	6	3	6	3	6	Zero divisor
4	4	2	6	4	2	6	Zero divisor
5	5	4	3	2	1	6	Unit
6	6	6	6	6	6	6	Zero

mod 10	1	2	3	4	5	6	7	8	9	10	Type
1	1	2	3	4	5	6	7	8	9	10	Unit
2	2	4	6	8	10	2	4	6	8	10	
3											
4	4	8	2	6	10	4	8	2	6	10	
5	5	10	5	10	5	10	5	10	5	10	Zero divisor
6											
7	7	4	1	8	5	2	9	6	3	10	
8	8	6	4	2	10	8	6	4	2	10	
9											
10	10	10	10	10	10	10	10	10	10	10	

3. Here is the start of the multiplication table:

mod 20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Type
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Unit
2	2	4	6	8	10	12	14	16	18	20	2	4	6	8	10	12	14	16	18	20	
3	3	6	9	12	15	18	1	4	7	10	13	16	19	2	5	8	11	14	17	20	
4	4	8	12	16	20	4	8	12	16	20	4	8	12	16	20	4	8	12	16	20	
5	5	10	15	20	5	10	15	20	5	10	15	20	5	10	15	20	5	10	15	20	Zero divisor

(a) List 3 different units mod 20.

(b) How do you tell if a number is a zero divisor / zero?

(c) How do you tell if a number is a unit?

(d) Notice that $3 \times 7 \equiv 1 \pmod{20}$. Find a number that goes to 5 (mod 20) after being multiplied by 7.

(e) Explain how to divide by 7 (mod 20)

4. Here is a way to find the pairs of units. Let's try mod 21. Pairs of units are numbers that multiply to 1. So we need to factor 1. Regular 1 does not factor. $1 + 21 = 22$ does factor. $2 \times 11 = 22 \equiv 1 \pmod{21}$ so 2 and 11 are pairs of units.

Sometimes we don't get anything new. We keep going and try to factor $1 + 21 + 21 = 43$, but 43 doesn't factor ($1 \times 43 = 43$ but $43 \equiv 1 \pmod{21}$ so we are just saying $1 \times 1 \equiv 1$).

(a) Keep going to find a pair of units with 4 and another pair with 8:

(b) Keep going to find a pair of units with 5:

(c) Can you find one for 10?