Hard Way

MA111-011 Exam 2 Name: Practice

1. For each number and modulus give 5 other numbers equivalent to it.
At least one must be negative. At least one must be larger than 100.

(a) 7 (mod 5)

12, 17, 22, 27, ..., 507

2, -3, -8, -13, ..., -93, -98, -103

(b) 7 (mod 10)

7, 17, 27, 37, 47, ..., 97, 107, 117, ...

-3, -13, -23, ..., -103, -203, ...

(c) 30 (mod 21)

30, 51, 72, 93, 114, ...

9, -12, -33, -54, ...

(d) 18 (mod 26)

18, 44, 70, 96, 122, ...

-8, -34

(e) Explain in your own words how you get answers to this problem.

I added and subtracted the modulus from the number.

10 adds of 5 = add 50, 100 adds of 5 = add 500

2. For each number problem give the **standard representative** of the answer.

(a) 105 (mod 10)

-100

5

(b) 40 (mod 37)

-37

3

(c) 78 (mod 10)

-70

8

(d) 375 (mod 37)

-370

5

(e) Explain in your own words how you get answers to this problem.

Ditto, but I tried to get a small, positive number.

Subtract 7 tens = Subtract 70. Subtract 100 thirty sevens = -370

3. For each arithmetic problem give the **standard representative** of the answer.

(a) $105 + 78 \pmod{10}$

$183 - 180 = \boxed{3}$

(b) $40 + 375 \pmod{37}$

$415 - 370 = 45$ $\quad 45 - 37 = 8$ $\quad \boxed{8}$

(c) $105 - 78 \pmod{10}$

$27 - 20 = \boxed{7}$

(d) $40 - 375 \pmod{37}$

$-335 + 370 = \boxed{35}$

(e) $105 \times 78 \pmod{10}$

$8190 - 8180$ $\quad \boxed{10}$

(f) $40 \times 375 \pmod{37}$

$15000 - 3700 = 11300$ $\quad 11300 - 3700 = 7600$ $\quad 7600 - 3700 = 3900$ $\quad 3900 - 3700 = 200$ $\quad 200 - 185 = 15$

$37 \times 5 = 185$

(g) Explain in your own words how you get answers to this problem.

This is the hard way. Do the arithmetic and subtract like crazy.

4. For each division problem give the **standard representatives** of all of the answers. If there are no answers, mention that, for example "there are no solutions." The shorthand $7 \div 5 \pmod{13}$ means the standard representatives of all numbers (like 4) that when multiplied by 5 give 7 as the result, at least mod 13 ($4 \times 5 = 20 \equiv 7 \pmod{13}$ so 4 works. No other numbers work, since 5 is a unit mod 13). Sometimes we wrote $7 \div 5 \pmod{13}$ as $\frac{7}{5} \pmod{13}$.

(a) $7 \div 2 \pmod{13}$

$\frac{7}{2}$ nope. $\frac{7+13}{2} = \frac{20}{2} = 10$

$\boxed{10}$

(b) $7 \div 10 \pmod{13}$

$\frac{7}{10}$ nope $\frac{7+13}{10} = \frac{20}{10} = 2$

$\boxed{2}$

(c) $15 \div 20 \pmod{13}$

$\frac{15}{20}$ nope $\frac{15+13}{20} = \frac{28}{20}$ nope, $\frac{28+13}{20} = \frac{41}{20}$ nope.

$\frac{41+13+13+13}{20} = \frac{80}{20} = \boxed{4}$

(d) $-14 \div 10 \pmod{17}$

$\frac{-14}{10}$ nope $\frac{-14}{10-17} = \frac{-14}{-7} = 2$ ☺

$\boxed{2}$ ↑ sneaky

(e) Explain in your own words how you get answers to this problem

I mostly just added the modulus to the numerator until it was divisible by the denominator,

On (d) I changed the denominator instead, and it worked well. Usually doesn't though. Lucky!

5. For each number and modulus classify it as a unit, zero, or zero divisor. Explain why.

(a) $3 \pmod{21}$

zero divisor $\frac{0}{3} = \frac{21}{3} = 7$

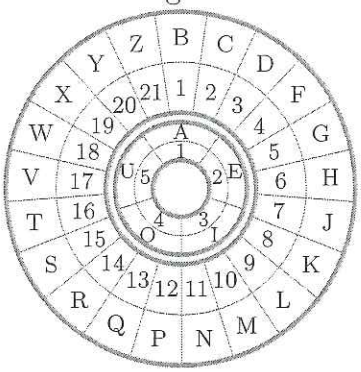
$3 \times 7 = 21 \equiv 0$

$2 \pmod{5}$

unit $\frac{1}{2} \equiv \frac{1+5}{2} = 3$

$2 \times 3 = 6 \equiv 1$

6. For each letter give 5 numbers equivalent to it. At least one must be negative. At least one must be larger than 100.



E $2, 7, 12, 17, 22, 27, \ldots, 507$
$-3, -8, -13, \ldots, -93, -98, -103$

L $9, 30, 51, 72, 93, 114, \ldots$
$-12, -33, -54, \ldots$

7. Label each letter with its number. Apply the shift by 12 cipher to the numbers, and then write down the corresponding letter. In other words, show your work for a shift encryption.

H o m e w o r k
6 4 10 2 18 4 14 8
18 1 1 4 9 1 5 20
W A B U L A N G Y

$+12 = -9 \mod 21$ (Consonants)
$+12 = +2 = -3 \mod 5$ (Vowels)

8. Label each letter with its number. Decrypt the shift by 12 cipher, first on the numbers, and then write down the corresponding letters. The result should be an encouraging word.

I L O H A B O
3 9 4 6 1 1 4
1, 18, 2, 15, 4, 10, 2
A W E S O M E

Reverse: $-12 = +9 \mod 21$
$-12 = -2 = +3 \mod 5$

Awesome!

9. What shift key was used to encrypt the word secret to the nonsense-word NOXMOP ?

s e c r e t
15 2 2 14 2 12
11 4 19 10 4 12
N O X M O P

Cons: $-4 = +17$
Vowel: $+2 = +7 = +12 = +17$
+17 works for both.

10. Copy the numbers from #7, and encrypt them with the multiply by 4 cipher, then write down the corresponding letters.

6 4 10 2 18 4 14 8
24 16 40 8 72 16 56 32
" " " " " " " "
3 1 19 3 9 1 14 11

DAXILARN

11. The following words were encrypted with the multiply by 4 cipher. Write down their numbers, divide them by 4, and write down the corresponding letter. The result should be an encouraging word.

~~TIQEZAEH~~

Wrong word →

~~16 3 13 2 21 1 2 6~~
" " " " " " " "
~~16 8 -8 12 0 -4 12 48~~

Right work →

$\div 4$: ~~4, 2, -2, 3, 0, -1, 3, 12~~
" " " "
19 21 4

~~FEXIZOIP~~

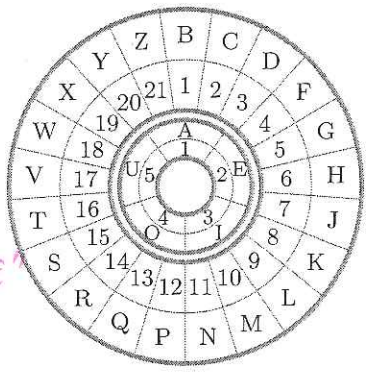
TOFUSAUW
16, 4, 4, 5, 15, 1, 5, 18
" " " " " " " "
0 36 -4 0 60

$\div 4$: 4, 1, 1, 0, 9, -1, 0, 15
" " " " " " " "
FABULOUS

12. Decrypt the following message. Show all work. If you are having trouble showing work, then write down some things you can tell about the message.

Hint: One of the words is "the" ← Con, Con, Vowel

OBU JEI DRU CYJ A OW VEETAXQ PEB?



4,1,5 . . . . . .

DRU is only CCVowel word. So it is "THE"

3 14 5

16 6 2

T H E

DRU → THE

Con +13 = -8 mod 21

Vow -3 = -8 = +2 mod 5

so key is -8 to decrypt, but C: +13 = -8 (two keys) V: -3 = +2 is OK.

O	B	U		J	E	I		D	R	U		C	Y	J		A		O	W
4	1	5		7	2	3		3	14	5		2	20	7		1		4	18
1	14	2		20	4	5		16	6	2		15	12	20		3		1	10
A	R	E		Y	O	U		T	H	E		S	P	Y		I		A	M
☺				☺				☺				☺ (no vowels!)				☺		☺	

V	E	E	T	A	X	Q		P	E	B	?
17	2	2	16	1	19	13		12	2	1	
9	4	4	8	3	11	5		4	4	14	
L	O	O	K	I	N	G		F	O	R	?

Are you the spy I am looking for?

Nice Way

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1. For each number and modulus give 5 other numbers equivalent to it.
At least one must be negative. At least one must be larger than 100.

(a) 7 (mod 5)

-8, 2, -3, 107, 112

(b) 7 (mod 10)

-13, 17, 27, 107, -3

(c) 30 (mod 21)

-33, 9, 51, -12, 114

(d) 18 (mod 26)

-34, -8, 44, 70, 96, 122

(e) Explain in your own words how you get answers to this problem.

I added and subtracted the modulus from the number.

2. For each number problem give the **standard representative** of the answer.

(a) 105 (mod 10)

5

(b) 40 (mod 37)

3

(c) 78 (mod 10)

8

(d) 375 (mod 37)

5

(e) Explain in your own words how you get answers to this problem.

I subtracted the modulus from the number until I got something between 1 and the modulus.

3. For each arithmetic problem give the **standard representative** of the answer.

(a) $105 + 78 \pmod{10}$

$5 + 8 \pmod{10}$
$= 13 \pmod{10} = \boxed{3}$

(b) $40 + 375 \pmod{37}$

$3 + 5 \pmod{37}$
$= \boxed{8}$

(c) $105 - 78 \pmod{10}$

$5 - 8 \pmod{10}$
$= -3 \pmod{10} = \boxed{7}$

(d) $40 - 375 \pmod{37}$

$3 - 5 \pmod{37}$
$= -2 \pmod{37} = \boxed{35}$

(e) $105 \times 78 \pmod{10}$

$5 \times 8 \pmod{10}$
$= 40 \pmod{10} = \boxed{10}$

(f) $40 \times 375 \pmod{37}$

$3 \times 5 \pmod{37}$
$= 15 \pmod{37} = \boxed{15}$

(g) Explain in your own words how you get answers to this problem.

I replaced each number with its standard representative. That made the arithmetic much easier!

4. For each division problem give the **standard representatives** of all of the answers. If there are no answers, mention that, for example "there are no solutions." The shorthand $7 \div 5 \pmod{13}$ means the standard representatives of all numbers (like 4) that when multiplied by 5 give 7 as the result, at least mod 13 ($4 \times 5 = 20 \equiv 7 \pmod{13}$ so 4 works. No other numbers work, since 5 is a unit mod 13). Sometimes we wrote $7 \div 5 \pmod{13}$ as $\frac{7}{5} \pmod{13}$.

(a) $7 \div 2 \pmod{13}$

$2 \times 10 = 20$

$20 \pmod{13} = 7 \pmod{13}$

So $7 \div 2 \pmod{13} =$ [10]

(b) $7 \div 10 \pmod{13}$

$10 \times 2 = 20$

$20 \pmod{13} = 7 \pmod{13}$

So $7 \div 10 \pmod{13} =$ [2]

(c) $15 \div 20 \pmod{13}$

$= \frac{15}{20} = \frac{3}{4}$

So this is like $3 \div 4 \pmod{13}$

$4 \times 4 = 16$

$16 \pmod{13} = 3 \pmod{13}$

So the answer is [4]

Check:

$20 \times 4 = 80$

$80 \bmod 13 = 2$ ✓

$15 \bmod 13 = 2$

(d) $-14 \div 10 \pmod{17}$

$= \frac{-14}{10} = \frac{-7}{5}$

So this is like

$-7 \div 5 \pmod{17}$

$= 10 \div 5 \pmod{17}$

$=$ [2]

Check:

$2 \times 10 = 20 \pmod{17}$

$= 3 \pmod{17}$ ✓

$= -14 \pmod{17}$

(e) Explain in your own words how you get answers to this problem

I looked for numbers where if I multiplied the second number by that number, I'd get the first number. Sometimes I looked at it like a fraction and reduced it to make it easier.

5. For each number and modulus classify it as a unit, zero, or zero divisor. Explain why.

(a) 3 (mod 21)

This is a zero divisor.

$3 \times 7 = 21 \pmod{21}$

$= 0 \pmod{21}$

2 (mod 5)

This is a unit. $2 \times 3 = 6 \pmod{5}$

$= 1 \pmod{5}$