Alex, Blair, Charlie, and Devin won a Halloween group costume contest and got a fabulous prize: vampire teeth (two top teeth, two bottom teeth). The four friends feel they each earned the teeth equally, but are not happy with the idea of each getting one tooth or having to take turns using the teeth. They want one person to get the teeth, but that person should pay the others for their share of the teeth.

In other words, right now four people each own 25% of the teeth, but they want one person to own 100% of the teeth and to refund the other people for their 25%.

Main Question: Who should get the teeth and how much should they pay each person?

There is a snag: the friends don't agree on how much the teeth are worth. Alex thinks they are worth \$10. Blair thinks they are worth \$12. Charlie thinks they are worth \$6. Devin thinks they are worth \$4. These are all what the friends think 100% of teeth are worth.

Each friend currently owns 25% of the teeth. We can calculate the 25% price from the 100% price: Alex won't take less than \$2.50 for their 25% of the teeth. Blair won't take less than \$3 for their 25% of the teeth. Charlie won't take less than \$1.50 for their 25% of the teeth. Devin won't take less than \$1 for their 25% of the teeth.

Whoever gets the teeth will need to purchase the other 75% of the teeth from their friends. We can calculate the 75% price from the 100% price: Alex won't pay more than \$7.50 total for the rest of the teeth. Blair won't pay more than \$9.00 total for the rest of the teeth. Charlie won't pay more than \$4.50 total for the rest of the teeth. Devin won't pay more than \$3.00 total for the rest of the teeth.

	100% Teeth	25% Teeth	75% Teeth
Alex	\$10	\$2.50	\$7.50
Blair	\$12	\$3.00	\$9.00
Charlie	\$ 6	\$1.50	\$4.50
Devin	\$ 4	\$1.00	\$3.00
Average	\$ 8	\$2.00	\$6.00

Main Question: Who should get the teeth, and who pays whom how much money?

Give a second solution:

Explain why you like one solution better than the other:

Another question: The four BFF roommates not only share the rent fairly, they also share boxes of breakfast cereal. However, relationships are strained currently as only three boxes of cereal included fabulous prizes. While the friends believe each friend should get 25% of the prizes, there are only three prizes, creating a bit of a difficulty.

	Sid toy	T-Rex toy	Tiana toy	Total	25% Total	75% Total
Alex	\$0.00	\$0.40	\$1.00	\$1.40	\$0.35	\$1.05
Blair	\$0.60	\$0.60	\$0.80	\$2.00	\$0.50	\$1.50
Charlie	\$0.80	\$0.80	\$0.80	\$2.40	\$0.60	\$1.80
Devin	\$1.00	\$1.00	\$0.80	\$2.80	\$0.70	\$2.10
Average	\$0.60	\$0.70	\$0.85	\$2.15	$\$0.53\frac{3}{4}$	$\$1.61\frac{1}{4}$

Who should get which toy and who should pay whom how much?

Give a second solution:

Explain why you like one solution better:

Is "Devin gets the toys and pays Alex, Blair, and Charlie each \$0.65" an ok solution?

How can you improve it?

Online Question: How do we solve these in general?

Imagine a fifth friend Frankie who owns 0% of the teeth, but has some cash on hand. Frankie pays everyone 25% for their teeth. He ends up paying $8 total ($2 on average tot he four friends). Now he wants to make money back, so he sells 100% of the teeth to one of the friends. Anyone who will pay him more than $8 is fine, but the bigger the better really. If Frankie sells to Alex, then Frankie gets $2 profit, and if Frankie sells to Blair, then Frankie gets $4 profit. If Frankie sells to Charlie or Devin, then Frankie suffers a loss (of $2 or $4 respectively).

Now we get rid of the imaginary friend Frankie and ask what the group can do as a group. The answer is the same. If the teeth are sold by the group to Alex, the group gets $2 profit (total, so only $0.50 on average). If Blair gets the teeth, the group gets $4 profit (total, so only $1 on average). If the teeth go to Charle or Devin, then the group suffers a loss of $2 or $4 total ($0.50 or $1 on average).

Answer: For each item calculate the average amount it is worth. The total profit for giving an item to someone is the difference between their 100% price and the average price. The average profit is the difference between their 25% price and the average 25% price. If you want people to get the most profit, then you just give the item to whoever wants it most. If for some reason that doesn't work, then you can give it to anyone who is willing to pay above average for it and still get a profit.