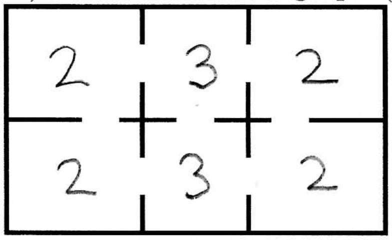
Practice Exam

Name: Key
MA111
2015-12-10

1. For each graph (or floorplan) label each vertex (or room) with its degree (number of doors). Label the whole graph (or floorplan) with the total number of edges (or doors).

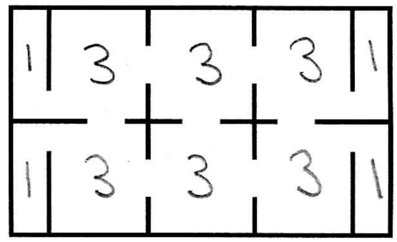
a) 

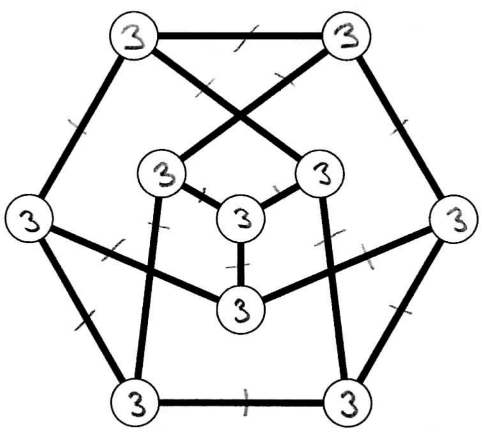
Total number of doors:

7

b) Total number of doors:

11



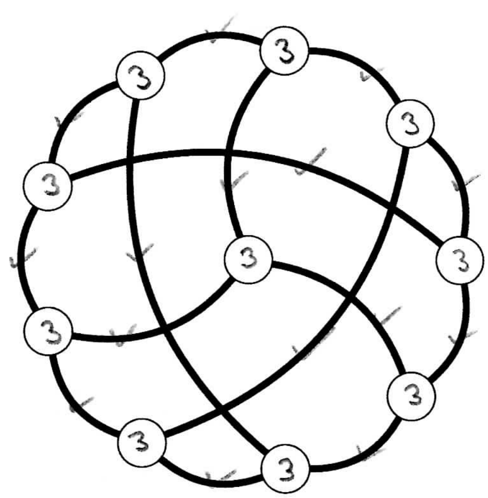
c) 

Total number of edges:

15

d) Total number of edges:

15

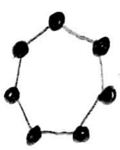


2. For each list of degrees below, calculate how many total edges (or doors) each graph (or floorplan) eould have and then draw an example graph (or floorplan) with those degrees. If it is impossible, then explain why.
Remember the yard counts as a room if you have exterior doors.

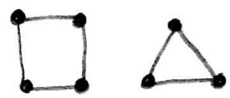
a) Degree 1, 1, 1, 1, 1, 1, 1 (seven 1s)

Not possible, because the sum of degrees is odd.
Each edge contributes 2 to the sum of degrees, so the sum must be even.

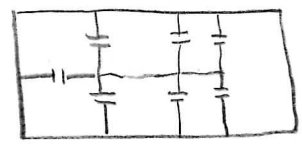
b) Degree 2, 2, 2, 2, 2, 2, 2 (seven 2s)



OR



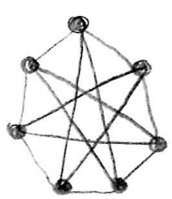
OR



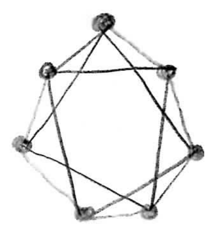
c) Degree 3, 3, 3, 3, 3, 3, 3 (seven 3s)

Not possible because the sum of degrees is odd.
Each edge contributes 2 to the sum of degrees, so the sum must be even.

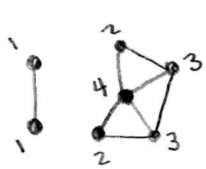
d) Degree 4, 4, 4, 4, 4, 4, 4 (seven 4s)



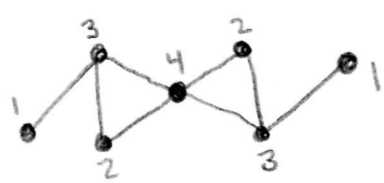
OR



e) Degree 1, 1, 2, 2, 3, 3, 4 (seven rooms)

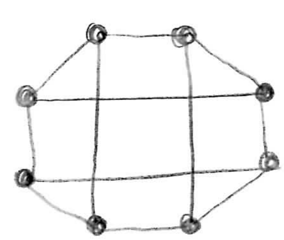


OR

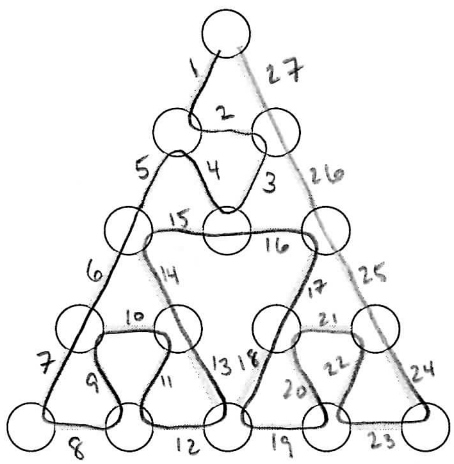


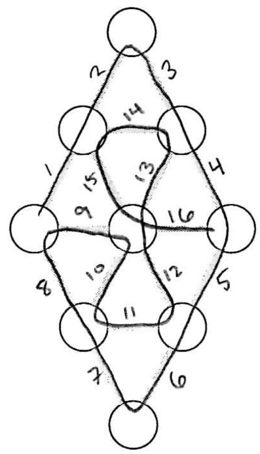
f) Degree 3, 3, 3, 3, 3, 3, 3, 3 (eight 3s)

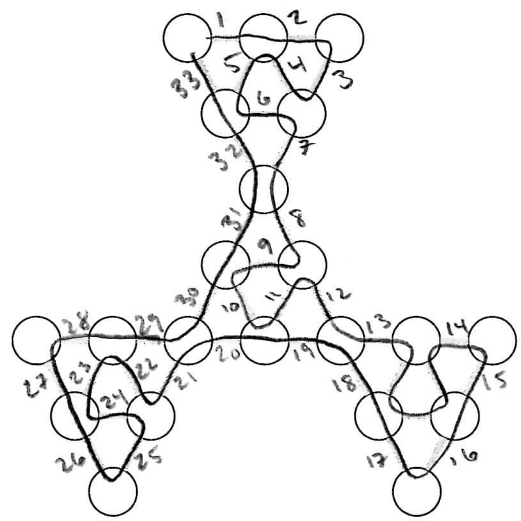
OR

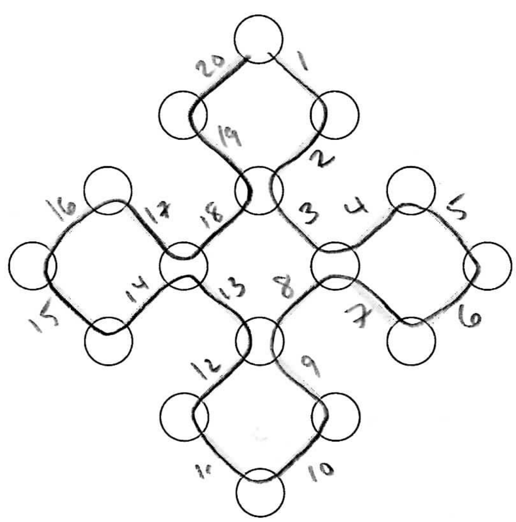


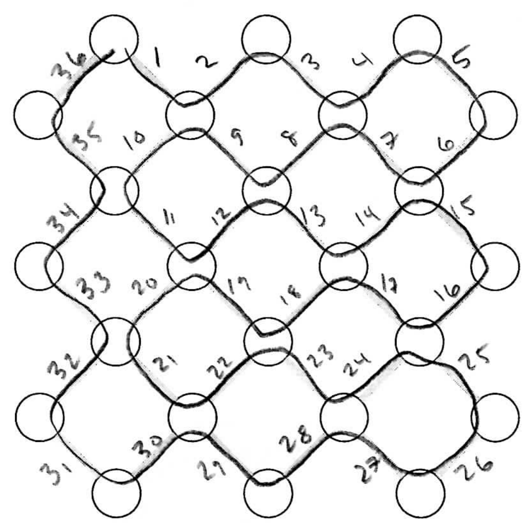
3. For each graph label the edges $1, 2, 3, \ldots$ in order of an Euler circuit or Euler path. You must clearly indicate the turns taken at each vertex. Illegible work receives no credit.











4. Label the degrees of each vertex, and then find optimal Eulerizations. Describe the Eulerization by darkening the edges that are repeated (don't add any truly new edges, only repeat old ones, until all the degrees are even). Each of the graphs is connected.