

# Advertise!\*

RICHARD EHRENBORG

These days it is not enough to teach interesting courses. We also have to attract students to take our courses. Even more so, we have to attract and retain students in the sciences and in mathematics.

I have been teaching a junior level course called *Applicable Algebra* at the University of Kentucky. The course begins with elementary number theory in order to cover topics including RSA, primality testing and factoring large integers. The course then continues to discuss polynomials in order to introduce finite fields and turn the attention to error correcting codes, such as BCH codes. The course focus is on the applications and how the mathematics supports them.

The course aims to make mathematics appealing to the students. The goal is twofold: First, to encourage students to major in mathematics. Second, to attract students from other majors, such as computer science, to double major in mathematics.

How does one get the word out to potential students? Advertise! Inspired by the NSA advertising campaign to hire mathematicians, I made a collection of posters. I handed them out to interested students and posted them around campus.

Here are the posters and some short explanations and references.

Shuffle a deck of cards eight times.  
Do it perfectly each time.  
Then you are back to where you started.  
Nice card trick.  
Why does it work?  
If you are curious, take CS 340/Math 340  
Spring 2006.

The perfect shuffles mentioned here are out-shuffles, that is, the top card stays on the top. For a deck with an even number  $n$  cards, label the positions 0 through  $n - 1$ . Now a perfect out-shuffle brings the card in position  $x$  to position  $2x \bmod n - 1$ . So what we need to verify is that  $2^8$  is congruent to 1 mod 51.

How do  
pineapples  
relate  
to the  
greatest  
common divisor?  
If you are curious...

Two consecutive Fibonacci numbers are the worst case scenario for Euclid's algorithm. Consecutive Fibonacci numbers also appear when counting spirals on pineapples and pine cones.

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What do  
cicadas  
know  
about  
prime  
numbers?

Assume that the cicadas appear every  $n$  years. A predator that appears every  $k$  years, where  $k$  is a divisor of  $n$ , can happily eat away. Hence the fewer divisors the period  $n$  has, the more likely the cicada population will survive. Prime numbers provide the minimum. See the article by Meredith Greer in the February 2006 issue of FOCUS.

Fermat  
thought  
4294967297  
was  
prime.  
He  
was  
wrong.

The number in question is  $2^{2^5} + 1$ . Euler showed that 641 is a prime factor. There is a nice way to do this using that  $641 = 2^7 \cdot 5 + 1 = 5^4 + 2^4$ .

How  
did  
the  
twin primes  
824633702441  
and  
824633702443  
improve  
your  
Pentium chip?

In the computation of Brun's constant, Tom Nicely discovered that the Pentium chip could not divide. See the nice article by Barry Cipra in *What is Happening in the Mathematical Sciences 1995–1996*, pages 38–47.

Why  
is  
the inequality  
 $|\ln(\text{lcm}(1, 2, \dots, n)) - n| \leq 2\sqrt{n}(\ln(n))^2$   
for  $n \geq 100$   
worth  
\$1,000,000?

This inequality is equivalent to the Riemann zeta hypothesis, something that we all should care about.

How are  
an old Greek,  
a French lawyer and  
a Swiss man  
involved in  
you sending  
your credit card number  
over  
the internet?

Fermat's little theorem and Euler's theorem are the basis of RSA, the first public key crypto system. Euclid also belongs since his algorithm allows us to find the decoding exponent from the encoding exponent and the secret factorization. I recommend the original article of R. L. RIVEST, A. SHAMIR AND L. ADELMAN, A method for obtaining digital signatures and public-key cryptosystems *Communications of the Association for Computing Machinery*, **21** (1978), no. 2, 120–126.

Your friend(?)  
claims that  
34034065601122854197959819122215174693  
is a prime number.  
How do you  
prove her/him  
wrong?

Primality testing is important when you need primes for your own RSA crypto system.

What is the probability  
that two people  
in this room  
have the same  
birthday?  
What does this  
have to do  
with  
factoring  
LARGE numbers?

For a year with  $P$  days and about  $\sqrt{\ln(4) \cdot P}$  persons, the probability is about fifty-fifty. The Pollard Rho factoring method uses this fact to run in  $O(P^{1/2}) \leq O(N^{1/4})$  time to factor  $N$ , where  $P$  is the smallest prime factor of  $N$ .

How come  
your cat  
can scratch  
your favorite cd  
and  
still  
it  
sounds  
great?

And:

How do you send  
pictures  
from Mars  
over  
noisy channels  
and still  
get  
a clear picture  
to give  
to CNN?

The answer to both of these questions is error correcting codes. It is amazing that your cd-player can lose 2.5mm of track and you will not hear the difference. Also note that NASA put high resolution cameras on the two Mars rovers in order to make both scientific observations and good PR for themselves.

The  
NEW  
engineering  
MATH:  
error correcting codes  
and  
public key cryptography.

These topics are what we mathematicians should be teaching our computer science and engineering students.

There are three people.  
On each person we put either  
a blue or red hat.  
Each person can see  
the color of the hats of the other two.  
At the same time they have to guess  
the color of their own hats.  
Each of them says red, blue or pass.  
If one of them is wrong, they lose.  
If at least one of them is right they win  
\$1,000,000.  
What is their best strategy?

This hat problem is related to Hamming codes, the first known class of error correcting codes. See Mira Bernstein's article in the November 2001 issue of FOCUS or the article by Joe Buhler, "Hat Tricks," in the Fall 2002 issue of *The Mathematical Intelligencer*.

Fun and interesting advertising is one way to attract more students to our classes. Finally, recall what Oscar Wilde wrote in *The Portrait of Dorian Gray*: There is only one thing in the world worse than being talked about, and that is not being talked about.

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