

Least squares approximation

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Find the least squares solution to the system

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

$$\text{Solution: If } A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \text{ then } A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 9 \end{bmatrix}, A^T A = \begin{bmatrix} 4 & 6 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix},$$

$$\text{and } A^T \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 27 \end{bmatrix}. \text{ So we are looking for solutions to the system}$$

$$\begin{bmatrix} 4 & 6 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix} \vec{x} = \begin{bmatrix} 6 \\ 11 \\ 27 \end{bmatrix}.$$

The matrices

$$\begin{bmatrix} 4 & 6 & 14 & 6 \\ 6 & 14 & 36 & 11 \\ 14 & 36 & 98 & 27 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 & \frac{9}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

are row equivalent, so the least squares solution is $\vec{x} = \begin{bmatrix} \frac{9}{10} \\ \frac{2}{5} \\ 0 \end{bmatrix}$. (And the quadratic best fit curve is linear!)