

- (1) Let $X \subset \mathbb{R}^n$ be the union of convex open sets X_1, \dots, X_n such that $X_i \cap X_j \cap X_k \neq \emptyset$ for all i, j, k . Show X is simply connected.
- (2) Show the complement of a finite set of points in \mathbb{R}^n is simply connected if $n \geq 3$.
- (3) The join of topological spaces X and Y , $X * Y$, is the quotient space of $X \times Y \times I$ under the identifications $(x, y, 0) \sim (x, y', 0)$ and $(x, y, 1) \sim (x', y, 1)$.

Show the join $X * Y$ of two nonempty spaces X and Y is simply connected if X is path connected.