# 9. Triangle Inequality for complex numbers

## 10. Multiplication of complex numbers

***Problem* 10.1.** Prove geometrically that $i \cdot z$ is $z$ rotated counterclockwise by 90 degrees.

***Problem* 10.2.** (HA) $[4, 2]$ is rotated counterclockwise by 90 degrees. Find the resulting vector.

**Hint(s) to 10.2**: What is the result of rotating $e_1$ and $e_2$ ? What is the result of rotating $a \cdot e_1 + b \cdot e_2$ ?

# **Answer to 10.2**: $[-2, 4]$

***Problem* 10.3.** Prove geometrically that $(\cos(\theta)+i\cdot\sin(\theta))\cdot z$ is $z$ rotated counterclockwise by $\theta$ radians.

***Problem* 10.4.** (HSA) The geometrical interpretation of $z \cdot (\cos(\alpha) + i \cdot \sin(\alpha))$ is $z$ rotated counterclockwise by angle $\alpha$. $[-7, 24]$ is rotated **clockwise** by 60 degrees. Find the resulting vector.

**Hint(s) to 10.4**: Clockwise rotation corresponds to negative angles.

# **Answer to 10.4**: $[17.28, 18.06]$

***Problem* 10.5.** Sketch a picture illustrating multiplication of unit complex numbers.

***Problem* 10.6.** (H) Show that $(3+4\cdot i)^{2004}+(3-4\cdot i)^{2004}$ is a real number.

**Hint(s) to 10.6**: Show that the number equals its conjugate.

***Problem* 10.7.** (H)

Prove that $z/(z^2+1)$ is a real number if $z$ lies on the unit circle.

**Hint(s) to 10.7**: Show that the number equals its conjugate. Use that $1/z = \bar{z}$ if $z$ is a unit complex number.

***Problem* 10.8.** (H)

Prove that $z/(z+1)^2$ is a real number if $z$ lies on the unit circle.

**Hint(s) to 10.8**: Show that the number equals its conjugate. Use that $1/z = \bar{z}$ if $z$ is a unit complex number.

***Problem* 10.9.** (H) Show that $i \cdot (1+z)/(1-z)$ is a real number if $z \neq 1$ is a unit complex number.

**Hint(s) to 10.9**: Show that the number equals its conjugate. Use that $1/z = \bar{z}$ if $z$ is a unit complex number.

***Problem*** **10.10.** (H) Show that $(1-z^2)\cdot w/(w^2-z^2)$ is a real number if $z \neq w$ are unit complex numbers.

**Hint(s) to 10.10**: Show that the number equals its conjugate. Use that $1/z = \bar{z}$ if $z$ is a unit complex number.

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