# *Math 460 Test 3 Friday July 1*

*Show your working!*

**1.** Let $\rho$ be the rotation of $\mathbb{R}^2$ given by

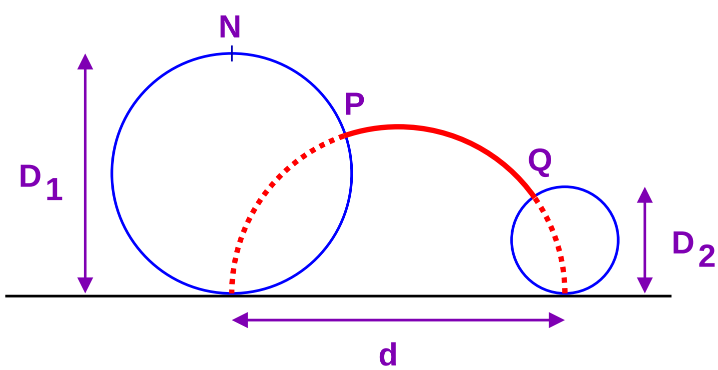
$$\rho\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} - 3 \\ \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} + 4 \end{bmatrix} .$$

Find the center and angle of the rotation $\rho$ .

**2.** Let $C_1$ be the circle of radius 2 centered at $(-3, 0)$ , and let $C_2$ be the circle of radius 2 centered at the point $(3, 0)$ . Let $\sigma$ , $\tau$ be inversions in the circles $C_1$ , $C_2$ respectively.

**(i)** Express each of $\sigma$ , $\tau$ in the form $z \mapsto \dfrac{a\overline{z} + b}{c\overline{z} + d}$ , and find expressions for $\sigma\tau(z)$ , $\tau\sigma(z)$ .

**(ii)** Determine the fixed points $P$ , $Q$ of $\sigma\tau$ . Explain briefly why $P$ , $Q$ are also the fixed points of $\tau\sigma$ .




**3.** *It is very important in this question to give adequate explanation for your results.* The diagram illustrates horocycles whose centers lie on the $x$-axis, Euclidean distance $d$ apart, and whose Euclidean diameters are $D_1$ , $D_2$ respectively. Also illustrated is a hyperbolic geodesic joining the centers of the two horocycles.

**(i)** Find the hyperbolic distance between the points $P$ , $Q$ where the geodesic meets the horocycles.

**(ii)** Find the hyperbolic length of the path along the horocycle of diameter $D_1$ from its “north pole” $N$ to the point $P$ .