Summary Information

Here, I have tried to gather the important information about the 3-D, 4-D, and even 5-D regulars, which are discussed in the preceding pages. Refer to this page often to clarify any of the key points. Also, this section should be very helpful for comparing various polytopes with one another, particularly noting the relationships between duals.

3D Regular Polytopes: The Platonic Solids

Tetrahedron:	4 vertices, 6 edges, 4 faces (triangles)					
Schläfli symbol:	$\{3,3\}$					
Coordinates:	$(1,1,1)$	$(1,-1,-1)$	$(-1,1,-1)$	$(-1,-1,1)$		
Octahedron:	6 vertices, 12 edges, 8 faces (triangles)					
Schläfli symbol:	$\{3,4\}$					
Coordinates:	$(1,0,0)$	$(0,1,0)$	$(0,0,1)$	$(-1,0,0)$	$(0,-1,0)$	$(0,0,-1)$
Cube:	8 vertices, 12 edges, 6 faces (squares)					
Schläfli symbol:	$\{4,3\}$					
Coordinates:	$(1,1,1)$	$(1,-1,1)$	$(-1,-1,1)$	$(-1,1,1)$		
	$(1,1,-1)$	$(1,-1,-1)$	$(-1,-1,-1)$	$(-1,1,-1)$		
Icosahedron:	12 vertices, 30 edges, 20 faces (triangles)					
Schläfli symbol:	$\{3,5\}$					
Coordinates:	$(0,\tau,1)$	$(1,0,\tau)$	$(\tau,1,0)$			
	$(0,-\tau,1)$	$(1,0,-\tau)$	$(-\tau,1,0)$			
	$(0,\tau,-1)$	$(-1,0,\tau)$	$(\tau,-1,0)$			
	$(0,-\tau,-1)$	$(-1,0,-\tau)$	$(-\tau,-1,0)$			
Dodecahedron:	20 vertices, 30 edges, 12 faces (pentagons)					
Schläfli symbol:	$\{5,3\}$					
Coordinates:	$(0,\tau-1,\tau)$	$(\tau,0,\tau-1)$	$(\tau-1,\tau,0)$	$(1,1,1)$	$(1,-1,-1)$	
	$(0,-\tau-1,\tau)$	$(\tau,0,-\tau-1)$	$(-\tau-1,\tau,0)$	$(1,1,-1)$	$(-1,1,-1)$	
	$(0,\tau-1,-\tau)$	$(-\tau,0,\tau-1)$	$(\tau-1,-\tau,0)$	$(1,-1,1)$	$(-1,-1,1)$	
	$(0,-\tau-1,-\tau)$	$(-\tau,0,-\tau-1)$	$(-\tau-1,-\tau,0)$	$(-1,1,1)$	$(-1,-1,-1)$	

Euler's Formula for Polyhedra: $V-E+F-2=0$

4D Regular Polytopes:

Simplex:	5 vertices, 10 edges, 10 faces, 5 cells (tetrahedra)	
Schläfli symbol:	$\{3,3,3\}$	
Coordinates:	$(-1,-1/\sqrt{3},-1/\sqrt{6},-1/\sqrt{10})$	$(1,-1/\sqrt{3},-1/\sqrt{6},-1/\sqrt{10})$
	$(0,2/\sqrt{3},-1/\sqrt{6},-1/\sqrt{10})$	$(0,0,3/\sqrt{6},-1/\sqrt{10})$
	$(0,0,0,4/\sqrt{10})$	

Hyperoctahedron:	8 vertices, 24 edges, 64 faces, 16 cells (tetrahedra)			
Schläfli symbol:	$\{3,3,4\}$			
Coordinates:	$(1,0,0,0)$	$(0,1,0,0)$	$(0,0,1,0)$	$(0,0,0,1)$
	$(-1,0,0,0)$	$(0,-1,0,0)$	$(0,0,-1,0)$	$(0,0,0,-1)$

Hypercube:	16 vertices, 32 edges, 24 faces, 8 cells (cubes)			
Schläfli symbol:	$\{4,3,3\}$			
Coordinates:	$(1,1,1,1)$	$(1,-1,1,1)$	$(-1,-1,1,1)$	$(-1,1,1,1)$
	$(1,1,-1,1)$	$(1,-1,-1,1)$	$(-1,-1,-1,1)$	$(-1,1,-1,1)$
	$(1,1,1,-1)$	$(1,-1,1,-1)$	$(-1,-1,1,-1)$	$(-1,1,1,-1)$
	$(1,1,-1,-1)$	$(1,-1,-1,-1)$	$(-1,-1,-1,-1)$	$(-1,1,-1,-1)$

24-cell:	24 vertices, 96 edges, 96 faces, 24 cells (octahedra)
Schläfli symbol:	$\{3,4,3\}$
Coordinates:	24 permutations of $(\pm 1,\pm 1,0,0)$

600-cell:	120 vertices, 720 edges, 1200 faces, 600 cells (tetrahedra)
Schläfli symbol:	$\{3,3,5\}$
Coordinates:	96 permutations of $(\pm\tau^{-1},\pm 1,\pm\tau,0)$
	8 permutations of $(\pm 2,0,0,0)$
	16 permutations of $(\pm 1,\pm 1,\pm 1,\pm 1)$

120-cell:	600 vertices, 1200 edges, 720 faces, 120 cells (dodecahedra)	
Schläfli symbol:	$\{5,3,3\}$	
Coordinates:	Permutations of:	Even permutations of:
	$(\pm 2,\pm 2,0,0)$	$(\pm\tau^2,\pm\tau^{-2},\pm 1,0)$
	$(\sqrt{5},\pm 1,\pm 1,\pm 1)$	$(\pm\sqrt{5},\pm\tau^{-1},\pm\tau,0)$
	$(\pm,\tau,\pm\tau,\pm\tau,\pm\tau^{-2})$	$(\pm 2,\pm 1,\pm\tau,\pm\tau^{-1})$
	$(\pm\tau^2,\pm\tau^{-1},\pm\tau^{-1},\pm\tau^{-1})$	

General Formula for Polytopes: $V-E+F-C=0$

Regular Polytopes:

Polytope	**Schläfli Symbol**	**V**	**E**	**F**	**C**
triangle	$\{3\}$	3	3	—	—
square	$\{4\}$	4	4	—	—
pentagon	$\{5\}$	5	5	—	—
hexagon	$\{6\}$	6	6	—	—
tetrahedron	$\{3,3\}$	4	6	4	—
octahedron	$\{3,4\}$	6	12	8	—
cube	$\{4,3\}$	8	12	6	—
icosahedron	$\{3,5\}$	12	30	20	—
dodecahedron	$\{5,3\}$	20	30	12	—
simplex	$\{3,3,3\}$	5	10	10	5
hyperoctahedron	$\{3,3,4\}$	8	24	32	16
hypercube	$\{4,3,3\}$	16	32	24	8
24-cell	$\{3,4,3\}$	24	96	96	24
600-cell	$\{3,3,5\}$	120	720	1200	600
120-cell	$\{5,3,3\}$	600	1200	720	120