MA 111 Review for Exam 5

1. Your parents purchased a government bond 18 years ago at $5\frac{1}{2}\%$ annual simple interest. Now it is worth \$6965. How much did your parents pay for the bond?

   Use $F = P(1+rt)$. Here, $R = 5.5\%$, $r = \frac{5.5}{100} = 0.055$, $t = 18$, $F = 6965$. So $6965 = P(1 + 0.055 \cdot 18) = P(1.99)$; $P = \frac{6965}{1.99} = \$3500$.

2. Suppose you deposit \$250 into a savings account with a 5% APR.

   (a) If the interest is compounded yearly, how much money will be in the account in 20 years?
   Use $F = P(1+\frac{r}{n})^{nt}$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 1$, $P = 250$, $t = 20$. So $F = 250(1+\frac{0.05}{1})^{1\cdot 20} = \$663.32$.

   (b) If the interest is compounded daily, how much money will be in the account in 20 years?
   Use $F = P(1+\frac{r}{n})^{nt}$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 365$, $P = 250$, $t = 20$. So $F = 250(1+\frac{0.05}{365})^{365\cdot 20} = \$679.52$.

3. Use the APY to answer the following: Which of the following investments is better: a 4% APR compounded monthly, or a 4.25% APR compounded annually?

   Check to see by what percent \$1 grows in one year in each case. Use $F = P(1+\frac{r}{n})^{nt}$. In the first case, $R = 4$, $r = \frac{4}{100} = 0.04$, $n = 12$, $P = 1$, $t = 1$. So $F = 1(1+\frac{0.04}{12})^{12} = \$1.0407$. This means that the APY is $\frac{1.0407-1}{1} \times 100 = 4.07\%$. In the second case, $R = 4.25$, $r = \frac{4.25}{100} = 0.0425$, $n = 1$, $P = 1$, $t = 1$. So $F = 1(1+\frac{0.0425}{1})^{1} = \$1.0425$. This means that the APY is $\frac{1.0425-1}{1} \times 100 = 4.25\%$. So the second investment is better.

4. A savings account at your bank offers a 4% APR compounded weekly.

   (a) If you make only one deposit of \$1500, and then don't touch the account, how much money will be in the account in 3 years?
   Use $F = P(1+\frac{r}{n})^{nt}$. Here, $R = 4\%$, $r = \frac{4}{100} = 0.04$, $n = 52$, $P = 1500$, $t = 3$. So $F = 1500(1+\frac{0.04}{52})^{52\cdot 3} = \$1691.17$.

   (b) If instead you deposit \$15 each week, how much money will be in the account in 3 years? (Assume that you deposit money at the beginning of the week, and interest is added at the end of the week.)

Use $F = L\left[\frac{(1+p)^T-1}{p}\right]$. Here, $R = 4$, $r = \frac{4}{100} = 0.04$, $n = 52$, $p = \frac{0.04}{52}$, $P = 15$, $L = 15(1+p) = 15(1+\frac{0.04}{52})$, $T = 52 \cdot 3 = 156$. So

$$\begin{aligned} F &= 15\left(1+\frac{0.04}{52}\right)\left[\frac{(1+\frac{0.04}{52})^{156}-1}{\frac{0.04}{52}}\right] \\ &= \$2487.09. \end{aligned}$$

5. Consider the geometric sequence with first two term $G_0 = 2$ and $G_1 = 5$.

   (a) Find the common ratio c.
   Note that $G_0 = P = 2$. Since $G_1 = cP$, then $5 = c \cdot 2$, so $c = \frac{5}{2} = 2.5$.

   (b) Give an explicit formula for G_N.
   $G_N = c^N P = (2.5)^N \cdot 2$.

   (c) Find G_7.
   $G_7 = (2.5)^7 \cdot 2 = 1220.70$.

   (d) Compute $G_0 + G_1 + G_2 + \cdots + G_{29} + G_{30}$.
   Use $P + cP + c^2P + \cdots + c^{N-1}P = P\left(\frac{c^N-1}{c-1}\right)$. So $2 + (2.5)2 + (2.5)^2 2 + \cdots + (2.5)^{30} 2 = 2\left(\frac{2.5^{31}-1}{2.5-1}\right) = 2.89 \times 10^{12}$.

6. You would like to save \$7,000 for an overseas trip that will begin in 18 months. If you plan to make a daily deposit into a savings account with an APR of 7%, compounded daily, how much should your daily deposit be in order to meet your goal? Assume there are exactly 30 days in each month, and that the last deposit generates no interest.
   Use $F = L\left[\frac{(1+p)^T-1}{p}\right]$. Here, $R = 7\%$, $r = \frac{7}{100} = 0.07$, $n = 365$, $p = \frac{0.07}{365}$, $F = 7000$, $T = 18 \cdot 30 = 540$, $L = P$ (because the last deposit generates no interest). So

$$\begin{aligned} 7000 &= P\left[\frac{(1+\frac{0.07}{365})^{540}-1}{\frac{0.07}{365}}\right] \\ 7000 &= P(568.895) \\ \frac{7000}{568.895} &= P \\ \$12.30 &= P. \end{aligned}$$

7. (a) Suppose you wish to purchase a car that costs \$11,500. If you can get a 10 year loan with a 5% APR compounded monthly, how much will your monthly payment be?

Use $P = Fq\left[\frac{q^T-1}{q-1}\right]$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 12$, $p = \frac{0.05}{12}$, $q = \frac{1}{1+\frac{0.05}{12}} = 0.995851$, $P = 11,500$, $T = 10 \cdot 12 = 120$. So

$$\begin{aligned} 11500 &= F(0.995851)\left[\frac{0.995851^{120}-1}{0.995851-1}\right] \\ 11500 &= F(94.2833) \\ \frac{11500}{94.2833} &= \$121.97 \end{aligned}$$

(b) With such monthly payments, how much interest will you pay over the 10 years?

Total of all payments: $121.97 \times 120 = \$14636.40$. Total interest paid: $14636.40 - 11500 = \$3136.40$.

8. Your bank approves you for a 30 year home loan with a 6% APR compounded monthly and monthly payments of \$800. What price can you afford for your new house?

Use $P = Fq\left[\frac{q^T-1}{q-1}\right]$. Here, $R = 6\%$, $r = \frac{6}{100} = 0.06$, $n = 12$, $p = \frac{0.06}{12} = 0.005$, $q = \frac{1}{1+p} = \frac{1}{1.005}$, $F = 800$, $T = 30 \cdot 12 = 360$. So

$$\begin{aligned} P &= 800\left(\frac{1}{1.005}\right)\left[\frac{\left(\frac{1}{1.005}\right)^{360}-1}{\frac{1}{1.005}-1}\right] \\ &= \$133,433.29. \end{aligned}$$