Intro to Contemporary Math

Sorted Edges Algorithm

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Agenda

- Hamiltonian Circuits and Sorted Edges Algorithm

Announcements

- Check the course website and syllabus for exam times and office hours
- Homework EC is due on Saturday

Sorted Edges Algorithm (1-3)

Step 1: Pick the cheapest (lowest weight) unused edge in the graph.
Step 2: Repeat Step 1, adding the cheapest unused edge to the circuit **unless**:
a) adding the edge would make **3 chosen edges connect at the same vertex**, or
b) adding the edge would **create a circuit that does not go through every vertex,**
In these cases, try again with the next cheapest edge.
Step 3: Repeat until the circuit is complete. **If the graph has n vertices, you pick n edges.**

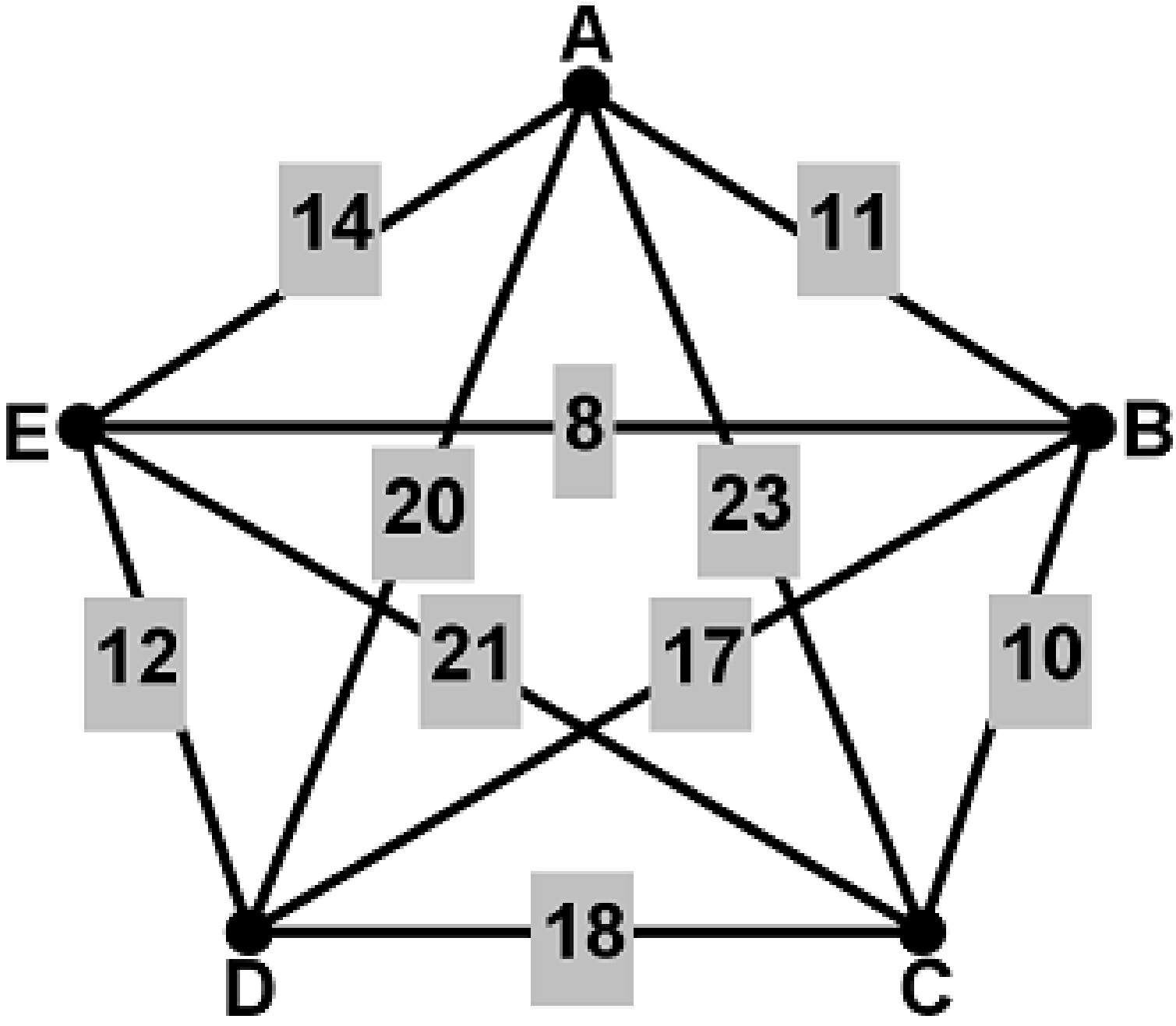
Sorted Edges Algorithm (2-Rec)

Step 2: Repeat Step 1, adding the cheapest unused edge to the circuit **unless**:
a) adding the edge would make **3 chosen edges connect at the same vertex**, or
b) adding the edge would **create a circuit that does not go through every vertex,**
In these cases, try again with the next cheapest edge.
Step 3: Repeat until the circuit is complete. **If the graph has n vertices, you pick n edges.**
Record your circuit by **listing the weights of the edges in the order they were chosen.**

Sorted Edges Demo

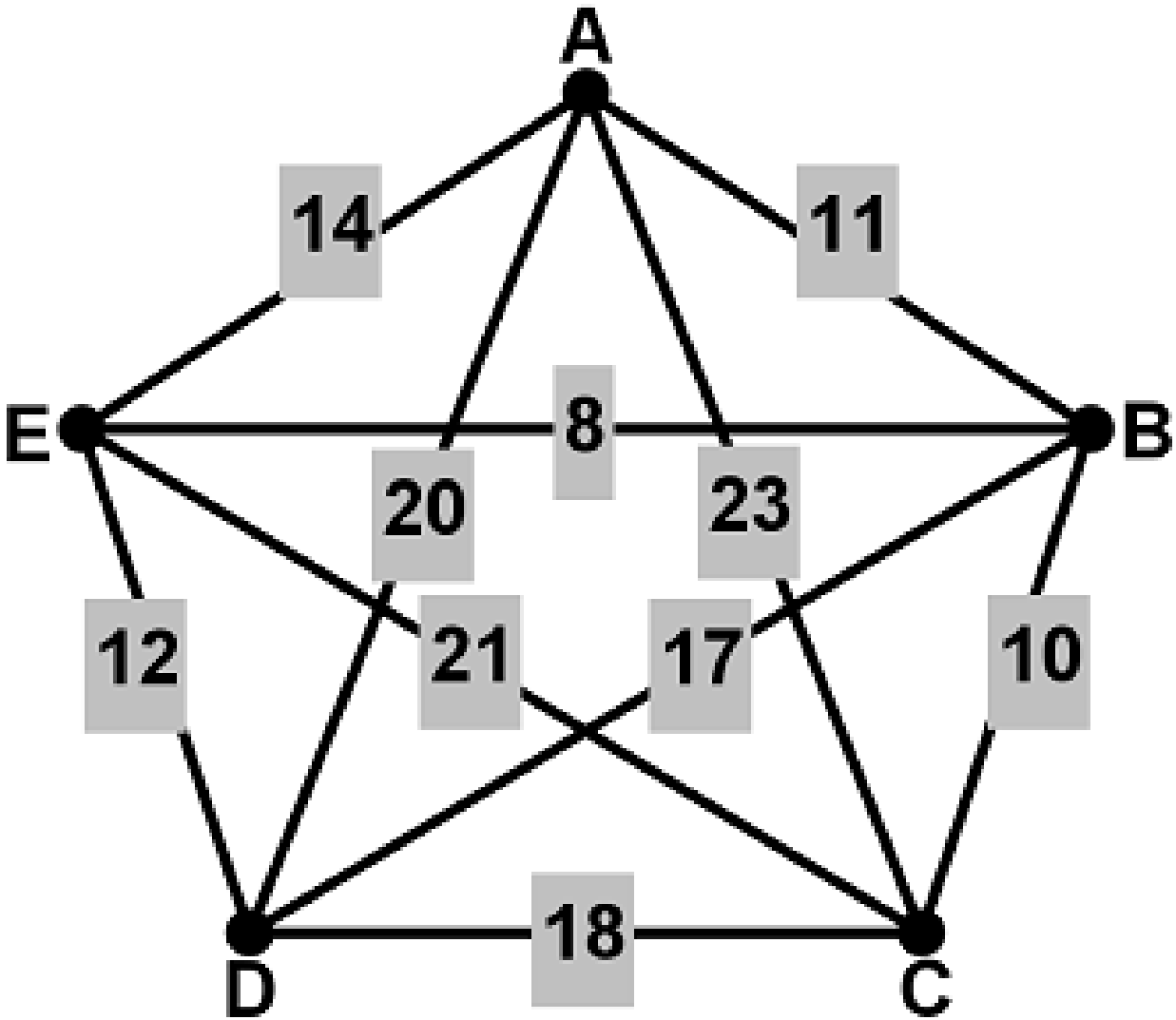
Use the Sorted Edges Algorithm. Record the edge weights in the order you pick them and find the total weight of the circuit.

Edge Weights: _____


A
14
11
E
8
B
20
23
12
21
17
10
18
D
C

?(8.1) First Edge?

What's the cheapest edge?

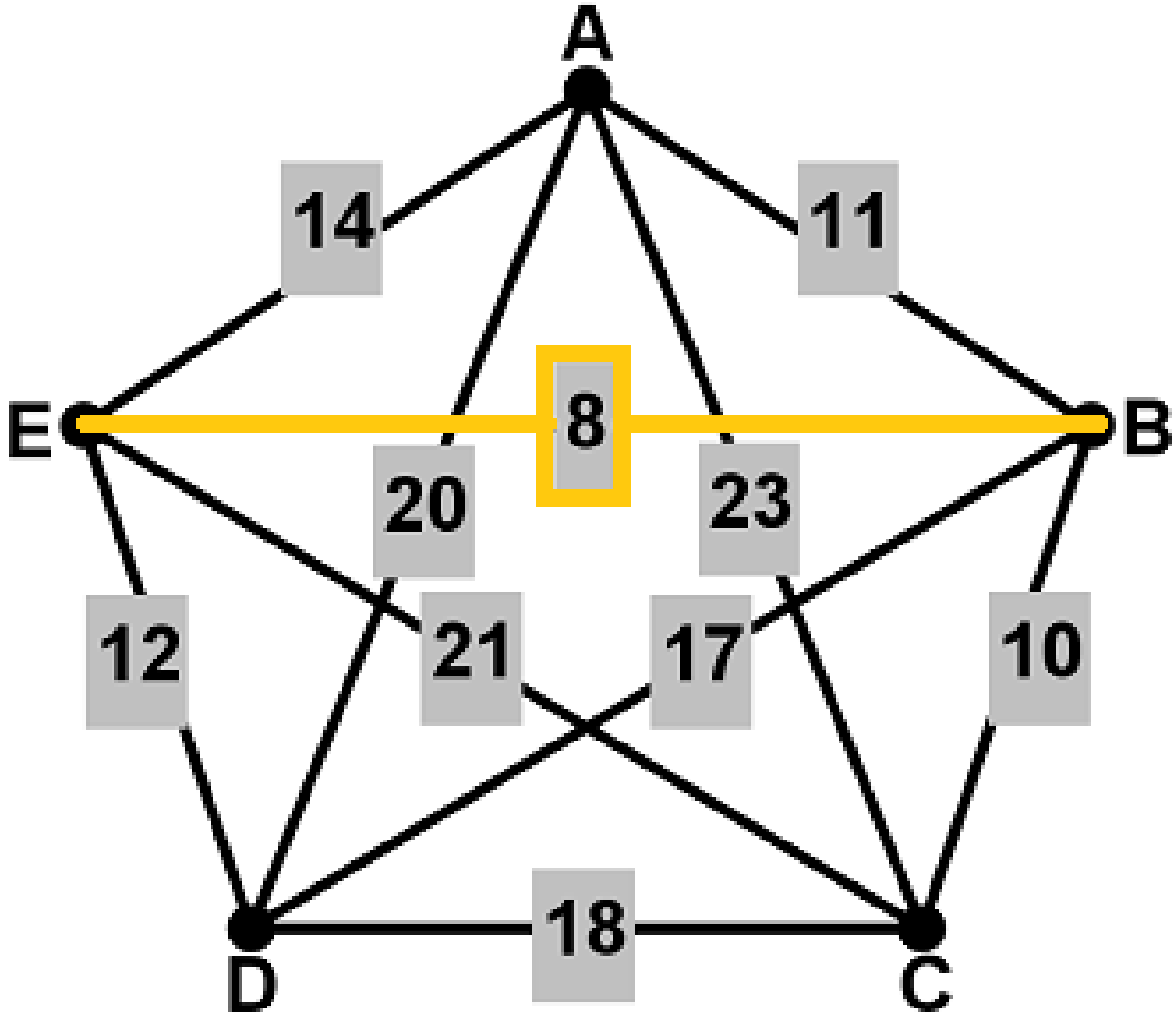


Type the edge's weight (number label)

Edges:

8
10
11
12
14
17
18
20
21
23

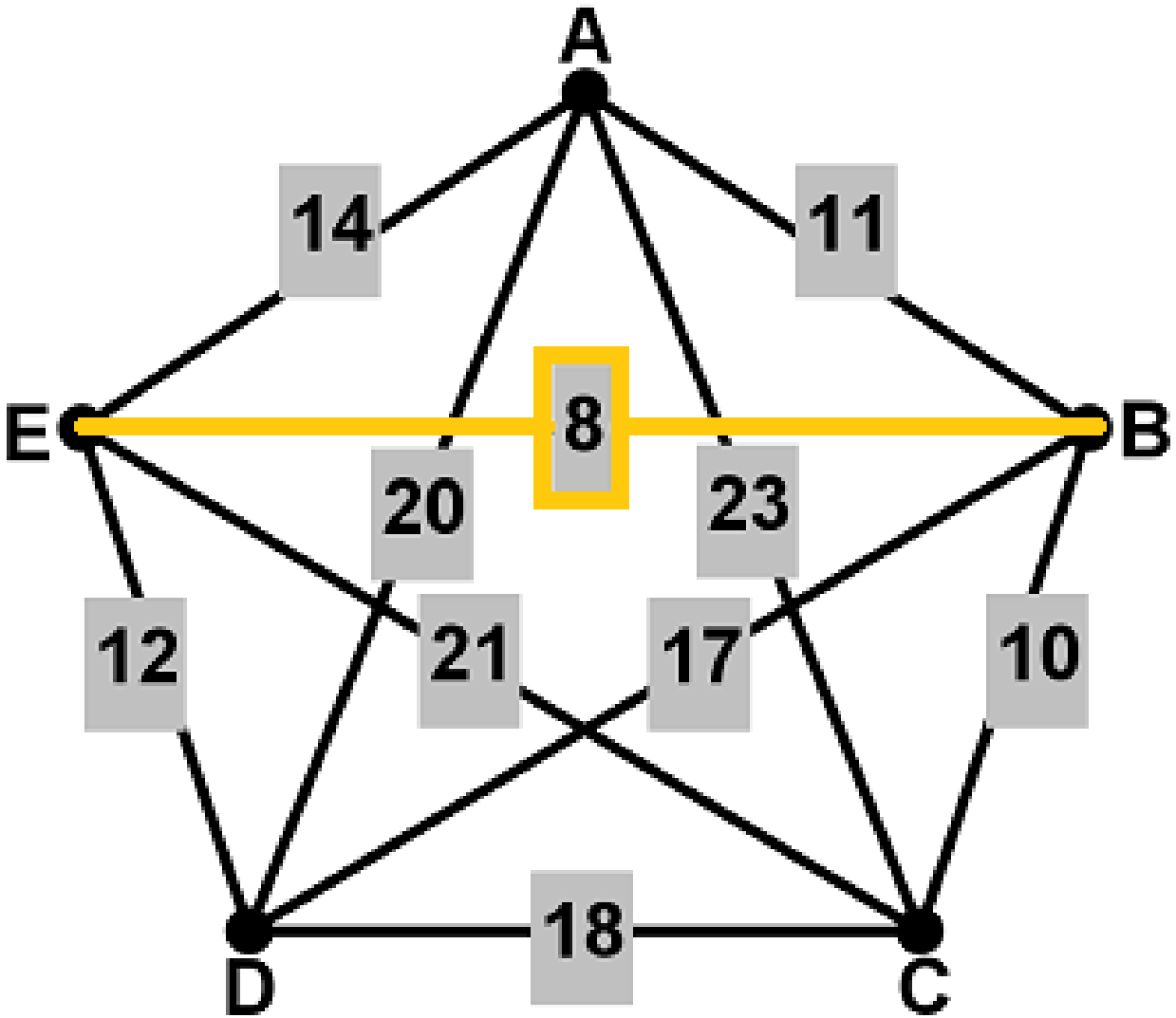
The 8 edge {B,E} is the cheapest edge. Pick it.



?(8.2) Next Edge?

Next edge?

Type the edge's weight (number label)



Edges:

~~8~~

10

11

12

14

17

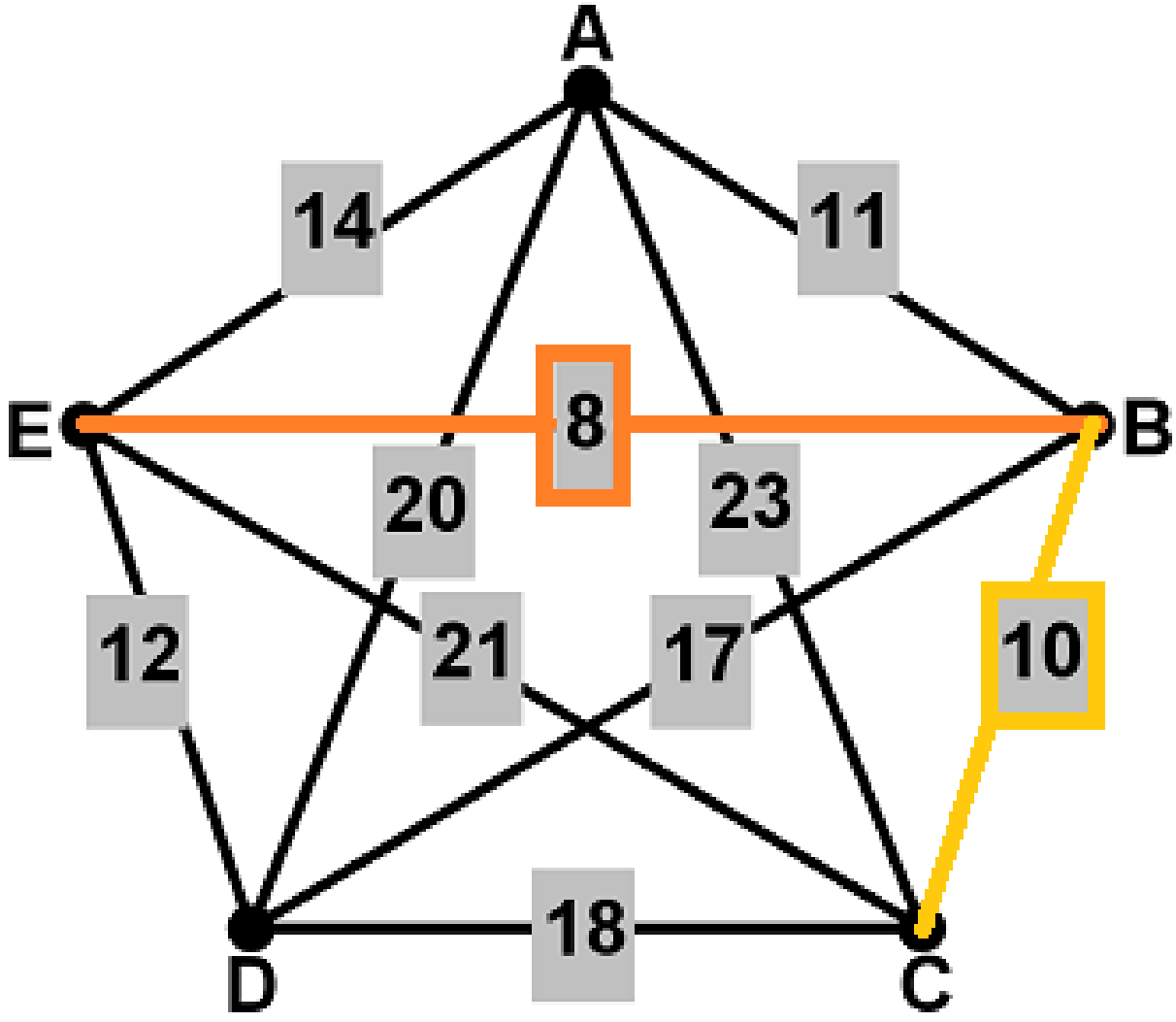
18

20

21

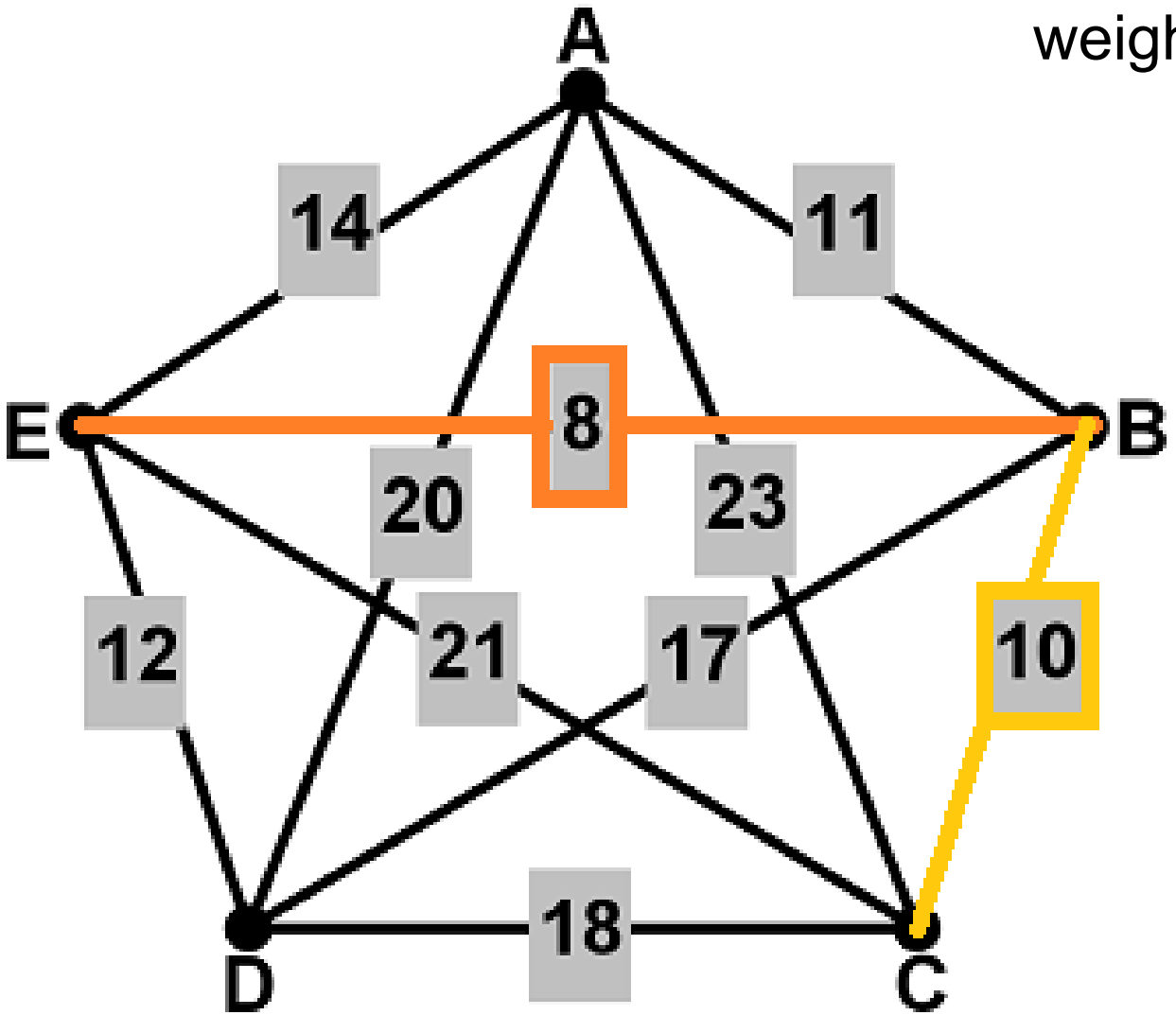
23

The next cheapest edge is the 10 edge {B,C}.



?(8.3) Next Edge?

Next edge? Warning: Look at Step 2a!



Type the edge's weight (number label)

Edges:

~~8~~

~~10~~

11

12

14

17

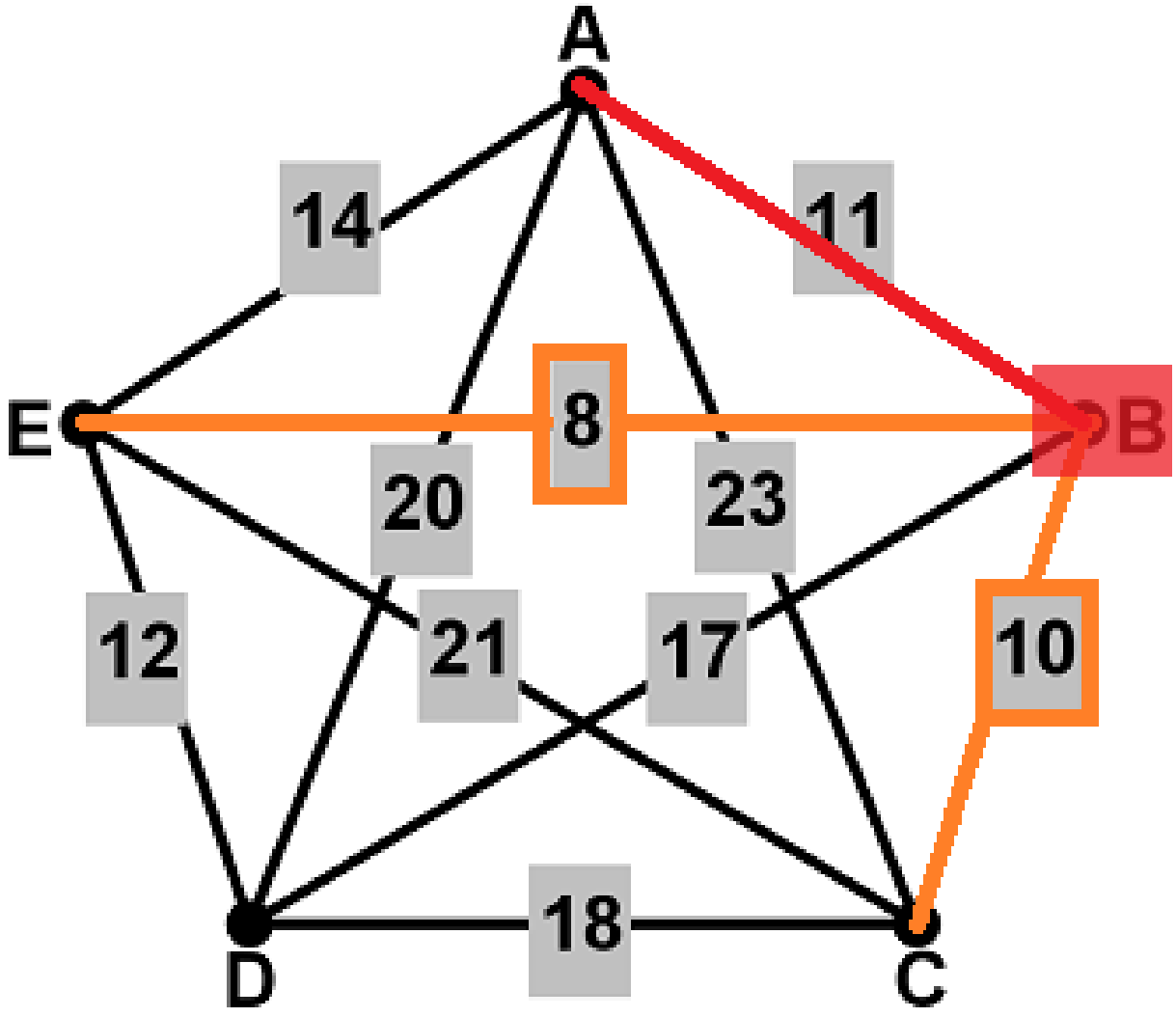
18

20

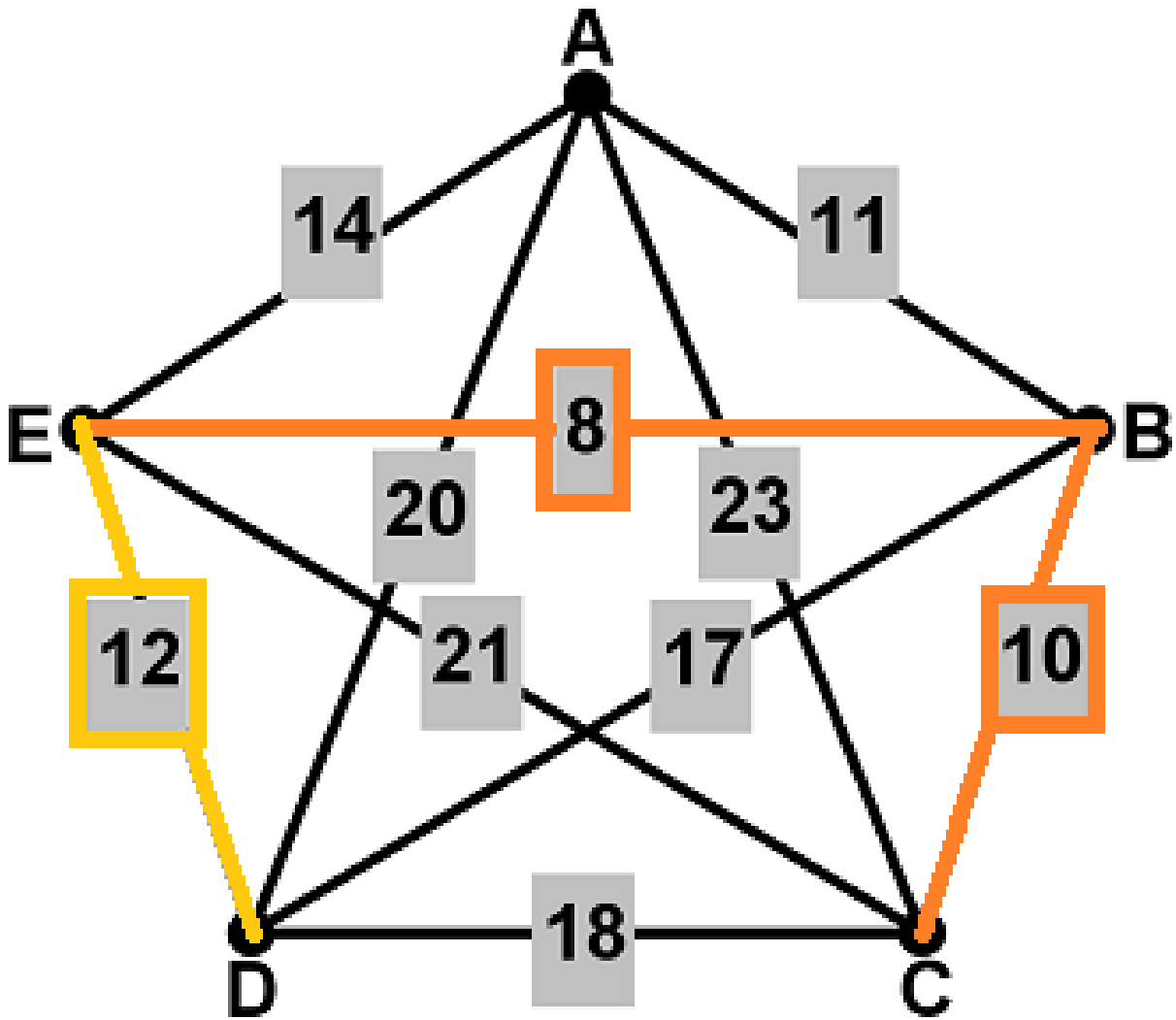
21

23

Do not pick 11 edge: causes 3 edges at B

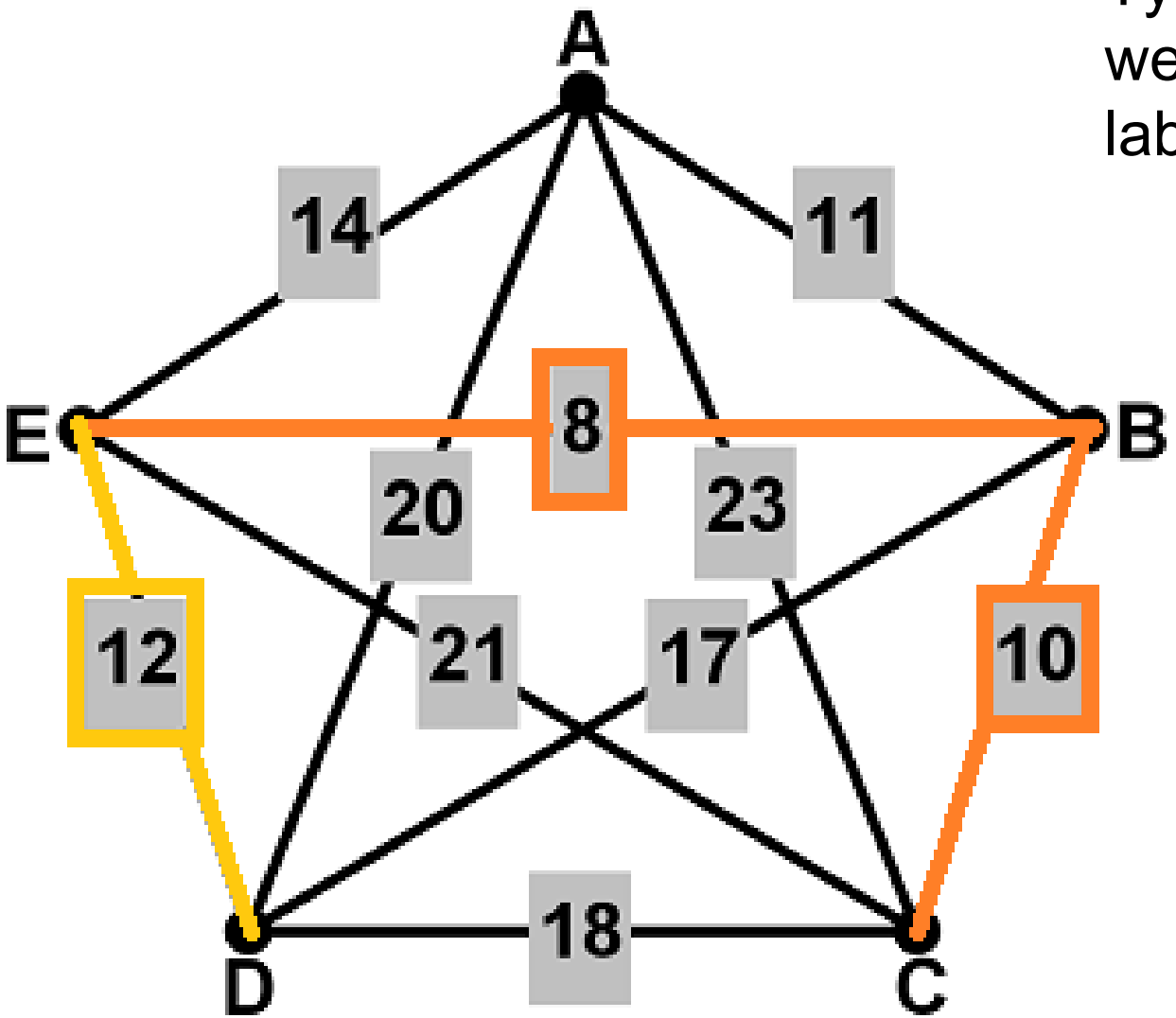


The next cheapest edge is the 12 edge {D,E}. We can jump around to find edges.



?(8.4) Next Edge?

Next edge? Warning: Look at Steps 2a and 2b!

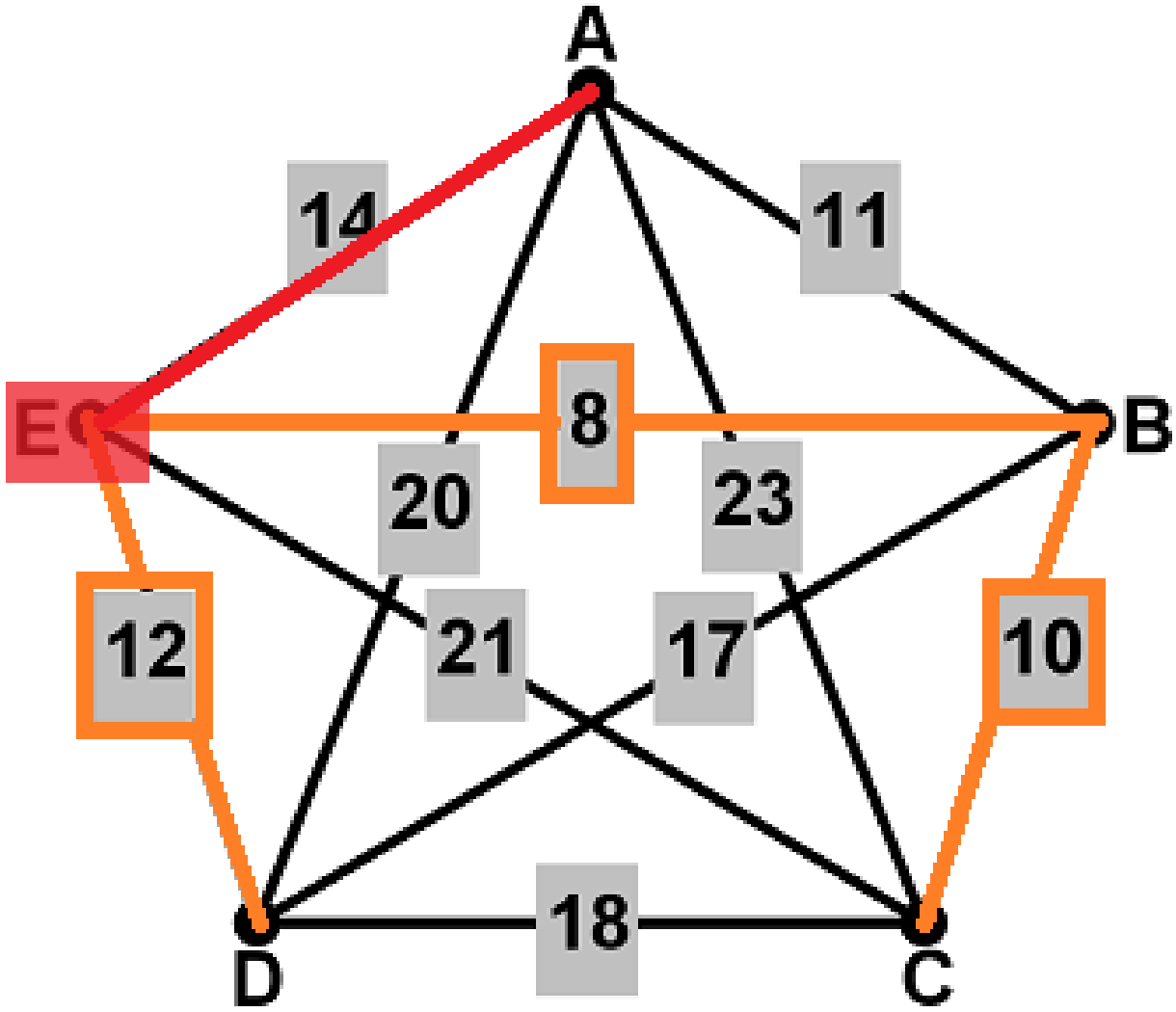


Type the edge's weight (number label)

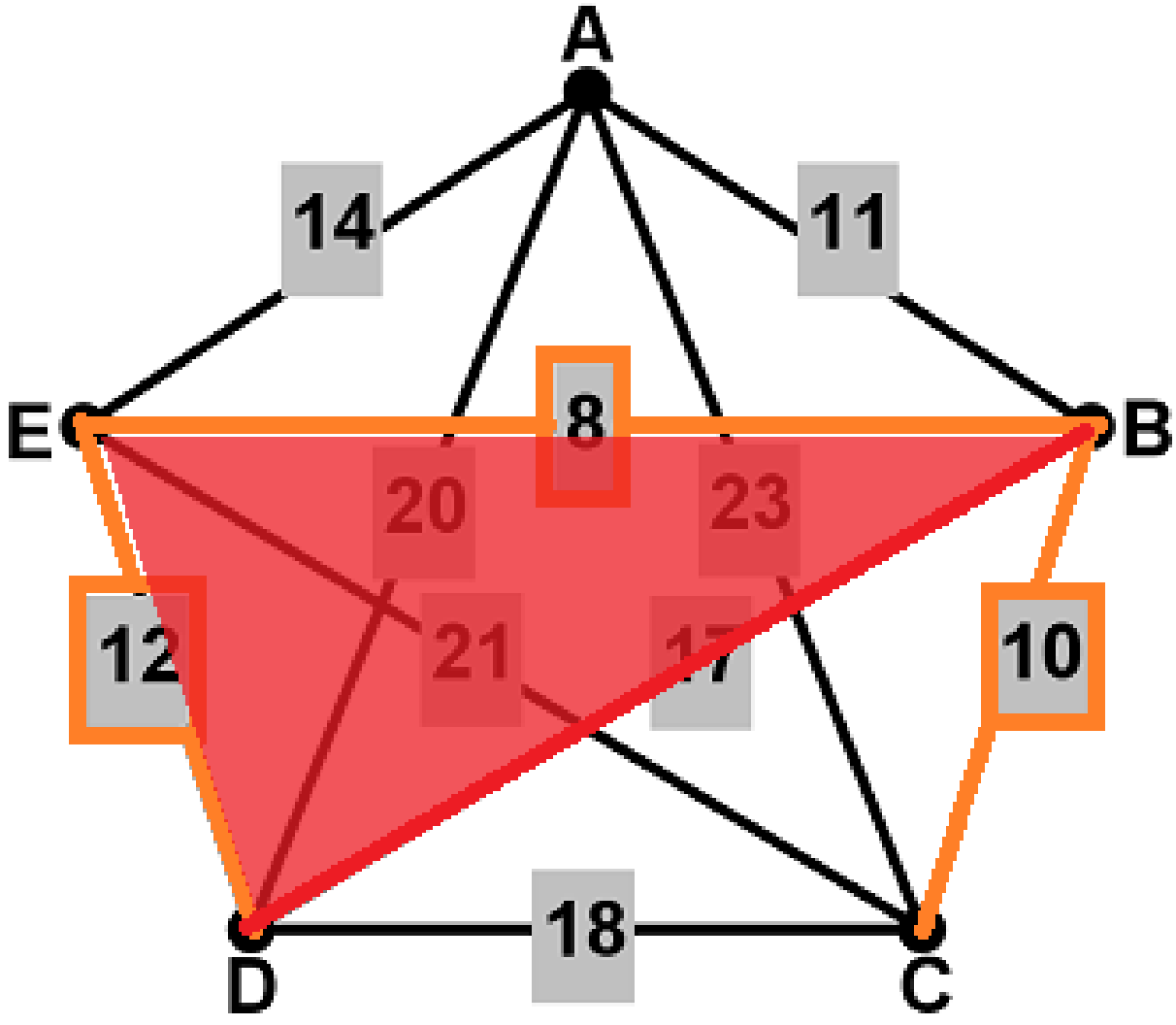
Edges:

~~8~~
~~10~~
~~11~~
~~12~~
14
17
18
20
21
23

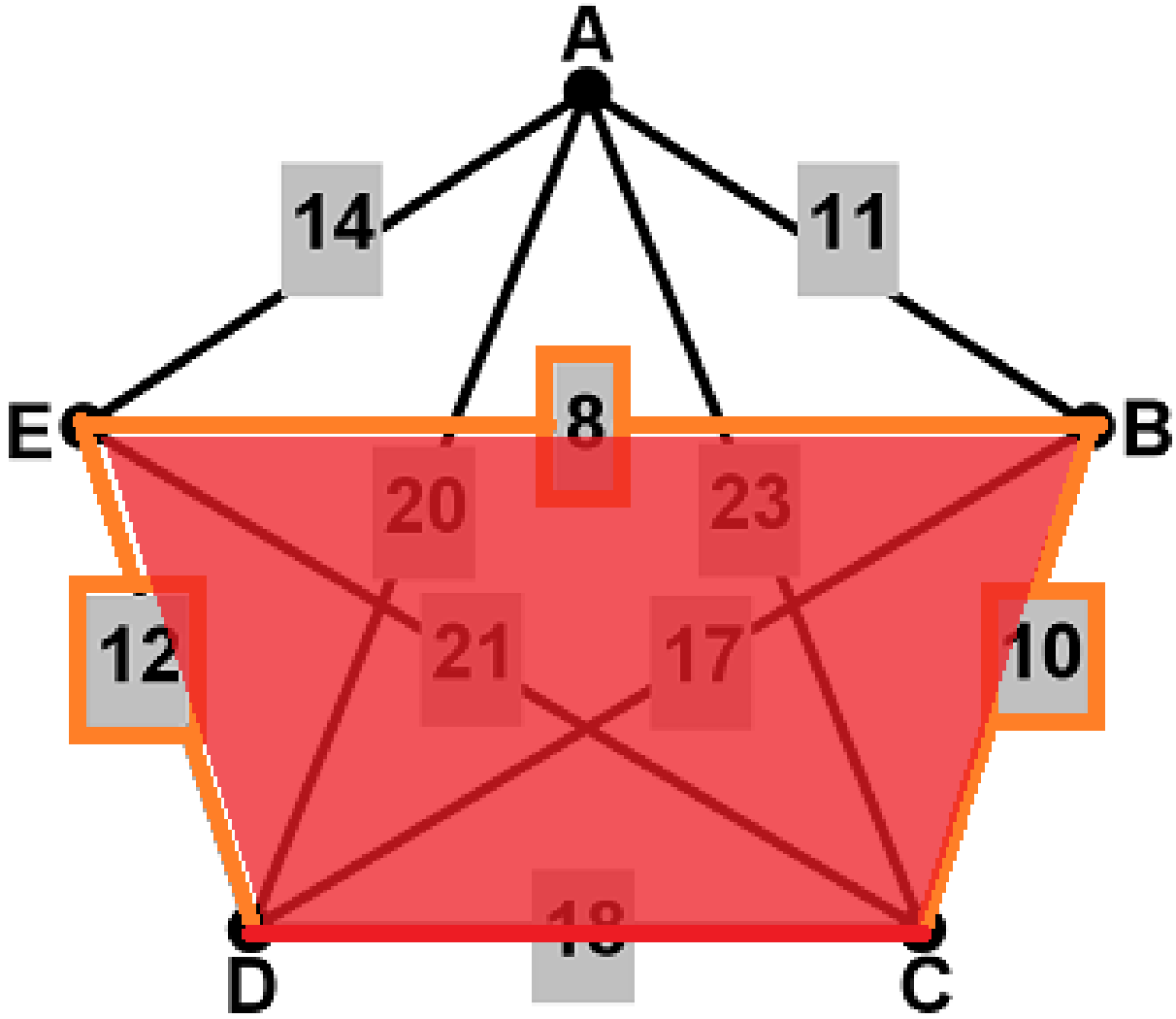
Do not pick 14 edge: 3 edges at E



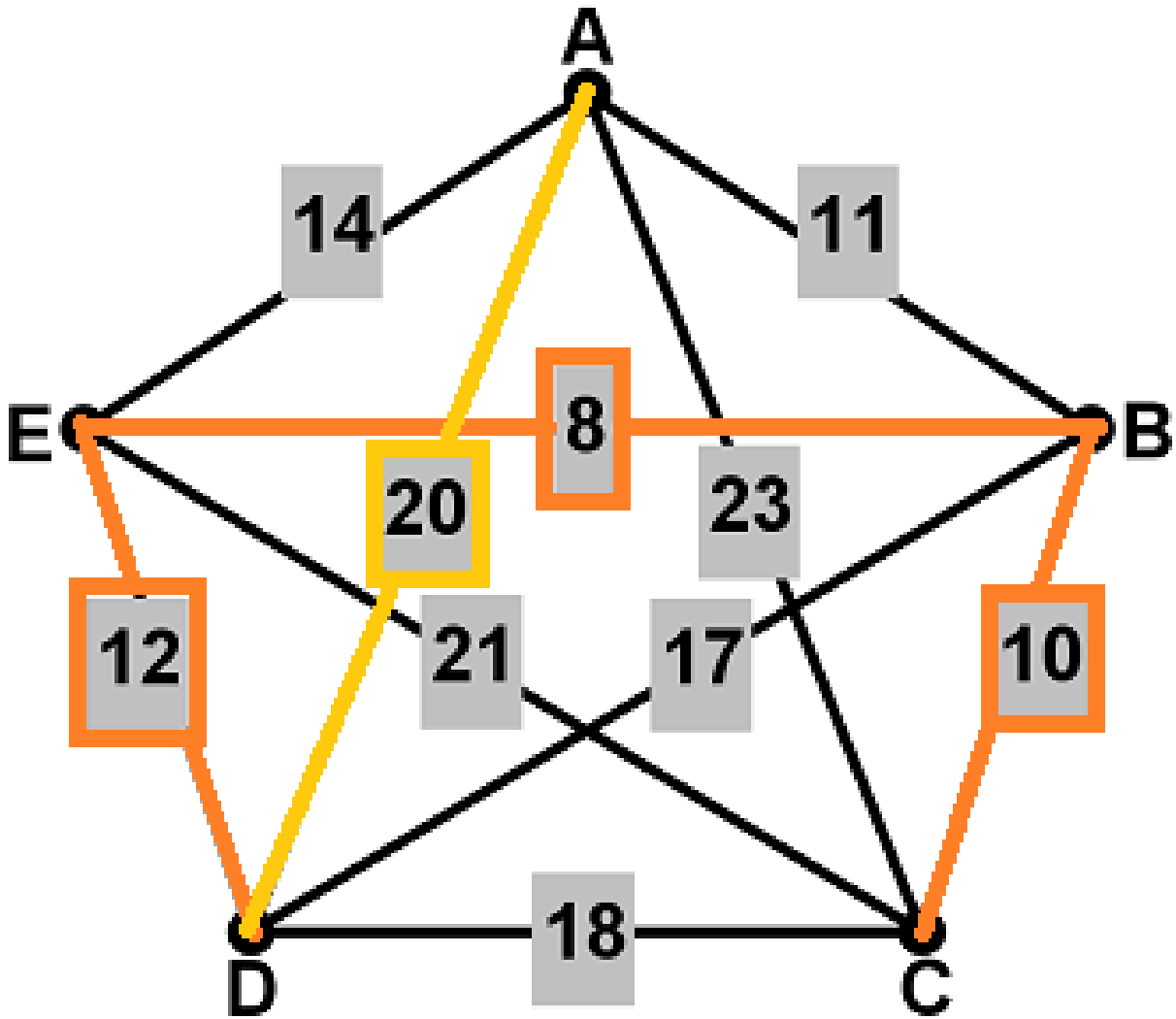
Do not pick 17 edge: makes circuit with B, D, E



Do not pick 18 edge: makes circuit with B, C, D, E

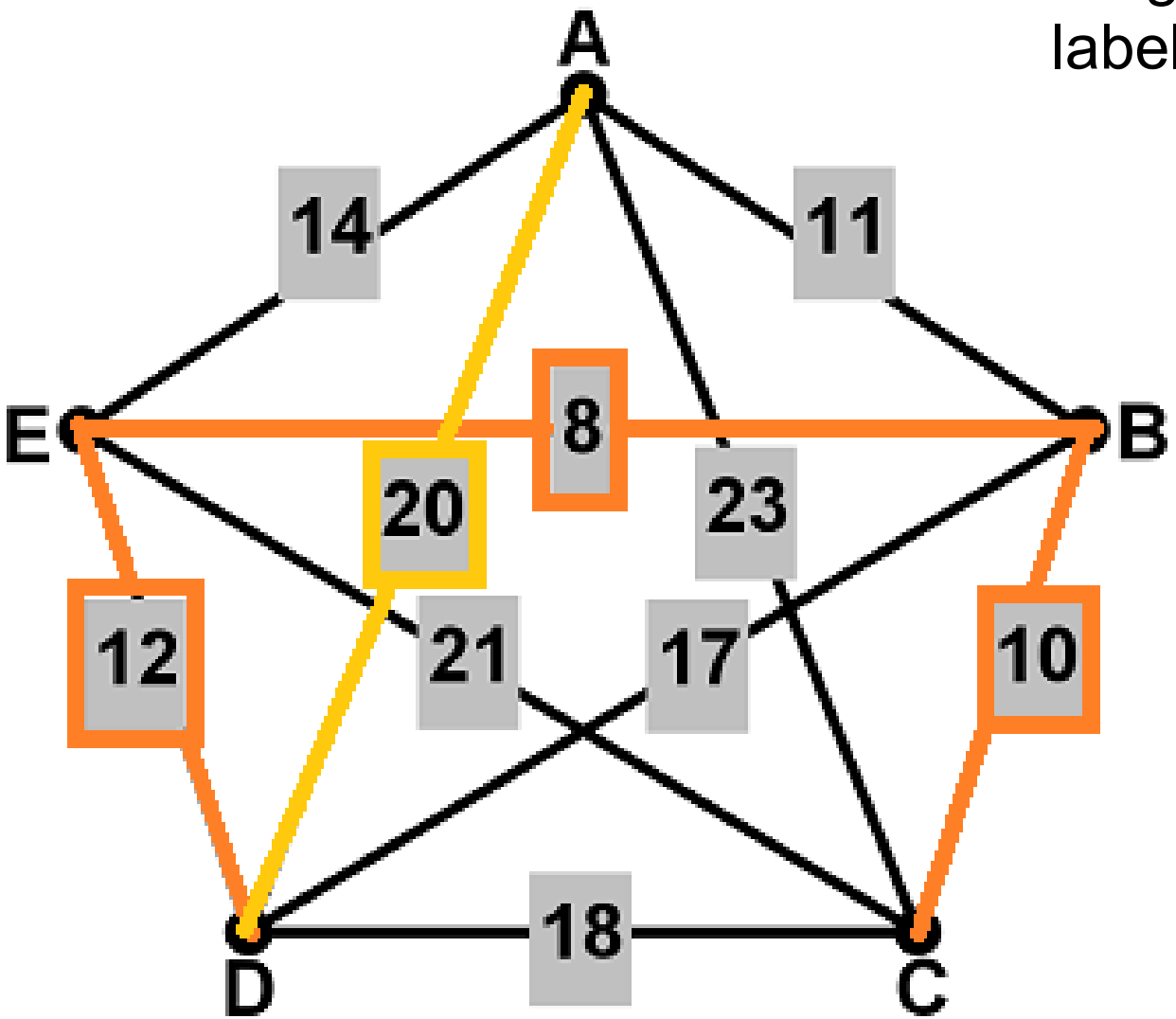


We have to pick the 20 edge {A,D} next. The 14, 17, and 18 edges were bad.



?(8.5) Next Edge?

Next edge?

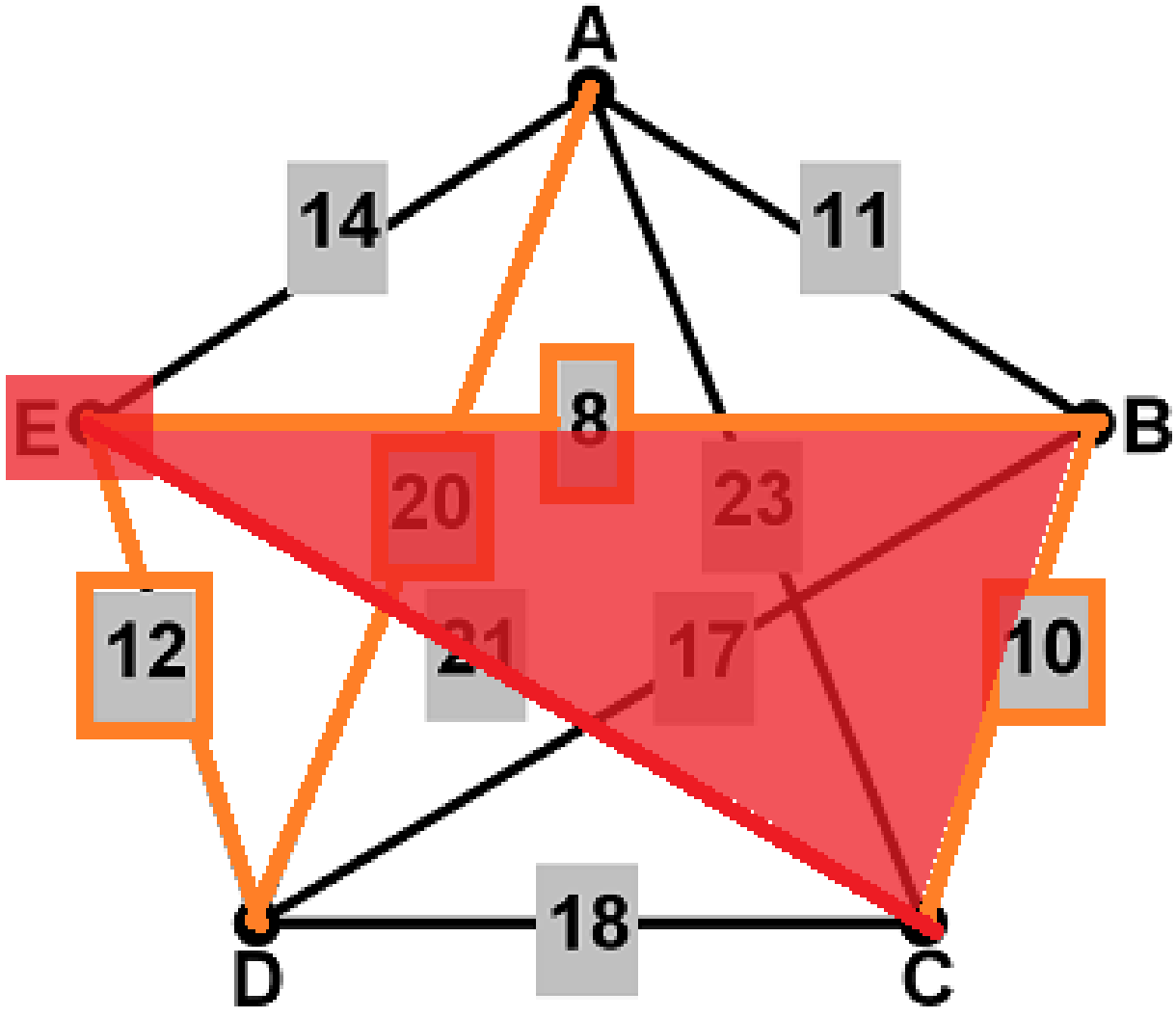


Type the edge's weight (number label)

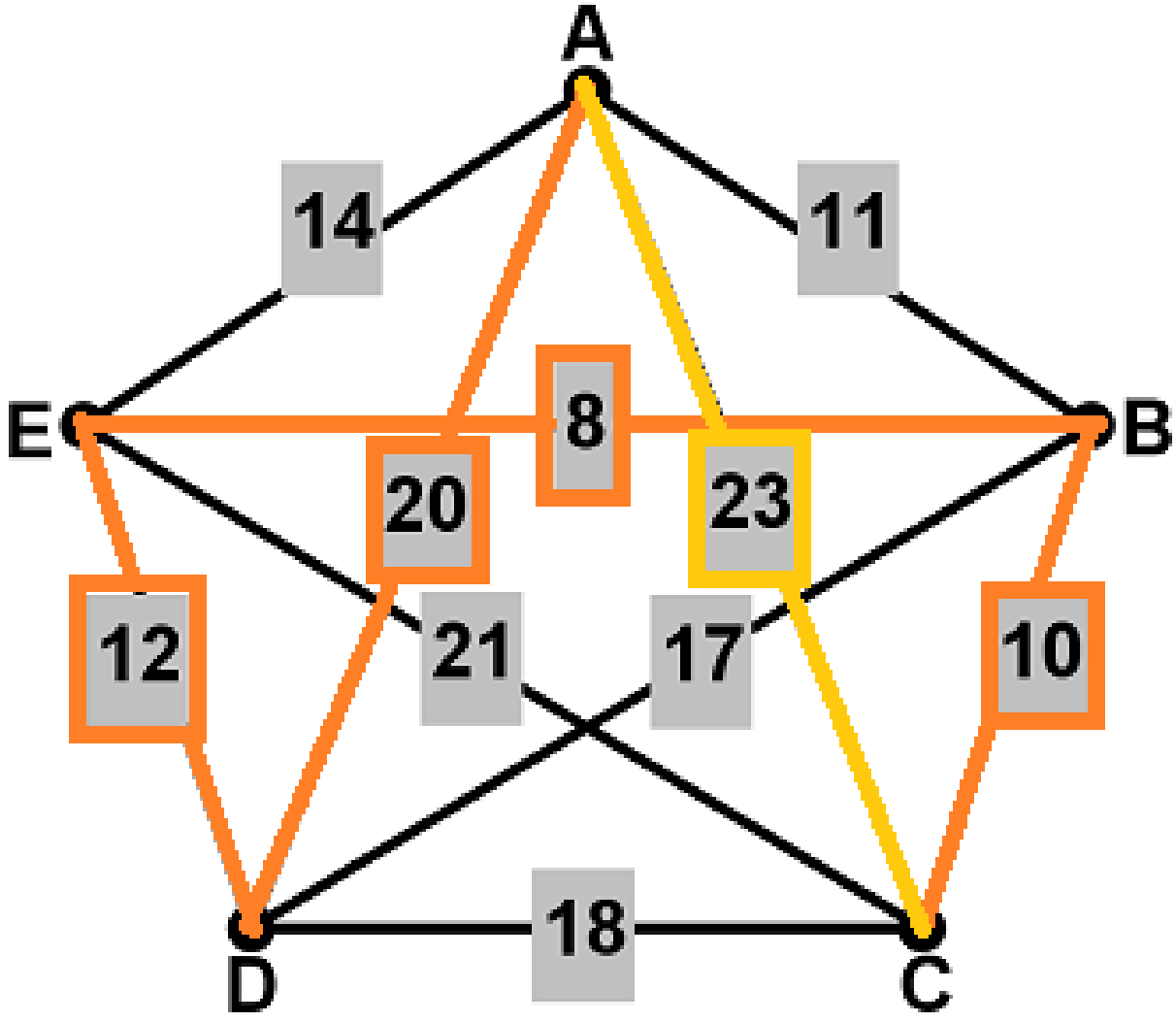
Edges:

~~8~~
~~10~~
~~11~~
~~12~~
~~14~~
~~17~~
~~18~~
~~20~~
21
23

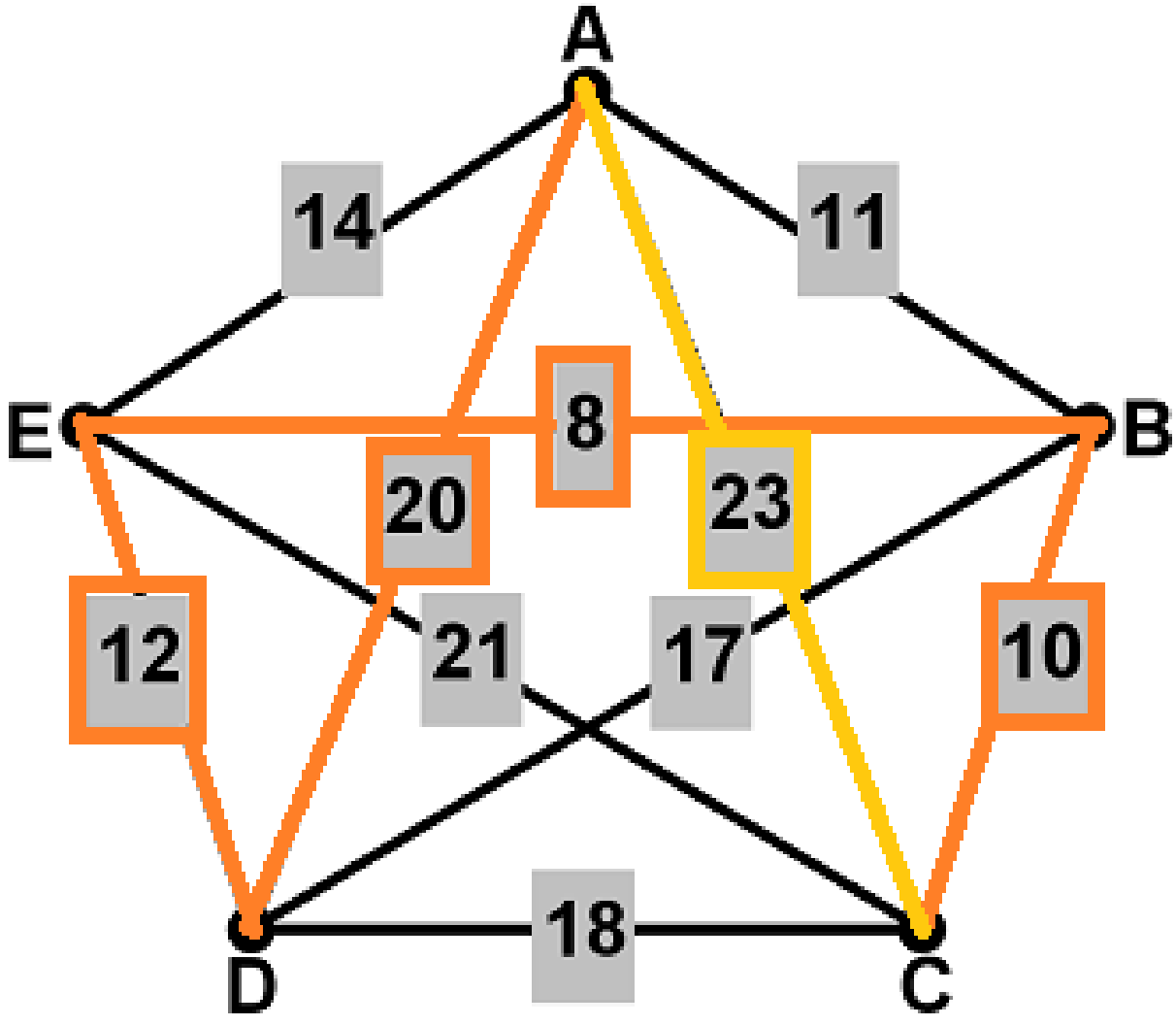
Do not pick 21 edge: 3 edges at E, and circuit B, C, E



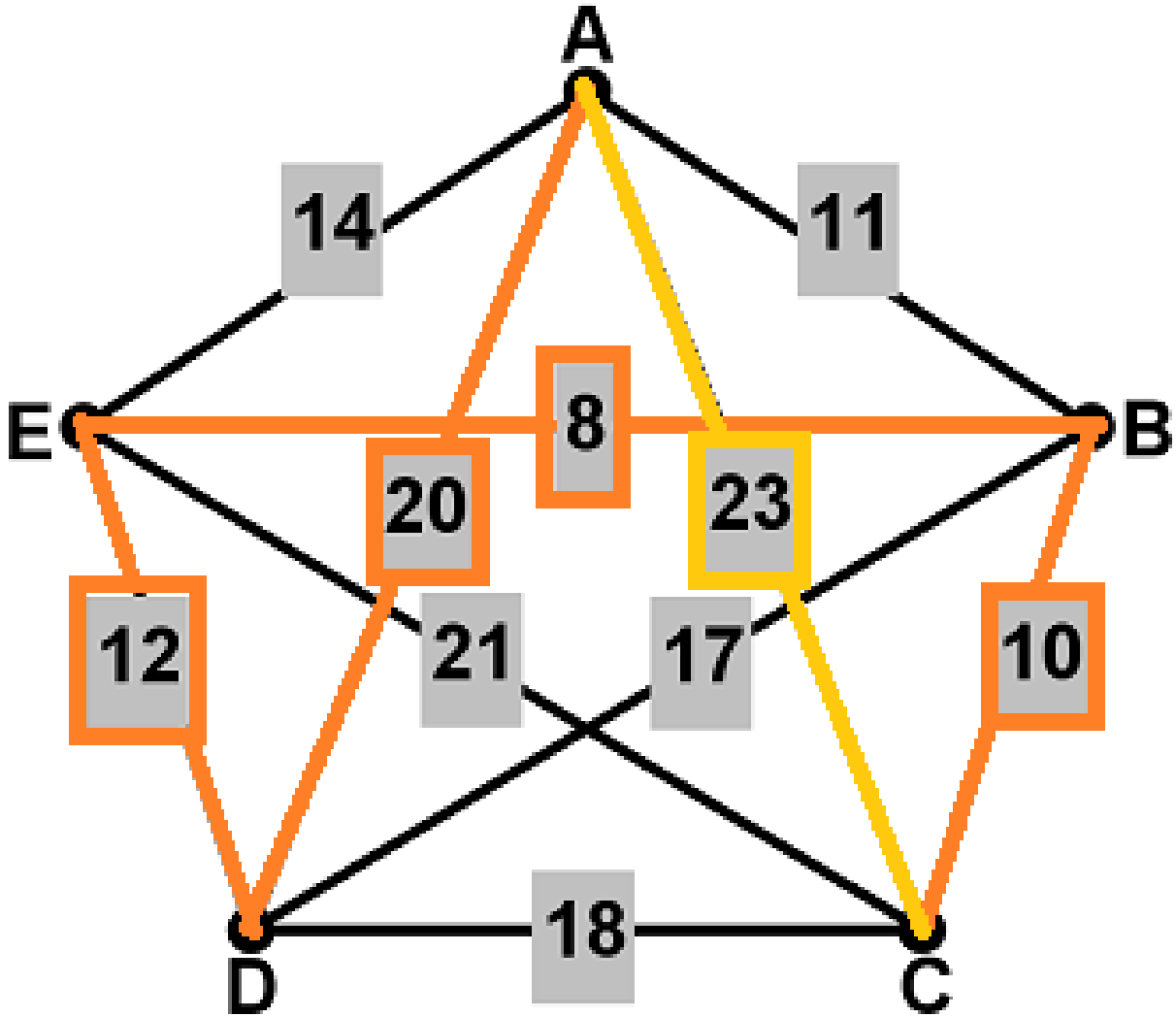
Pick 23 edge {A,C}.



Done: 5 vertices, so need 5 edges

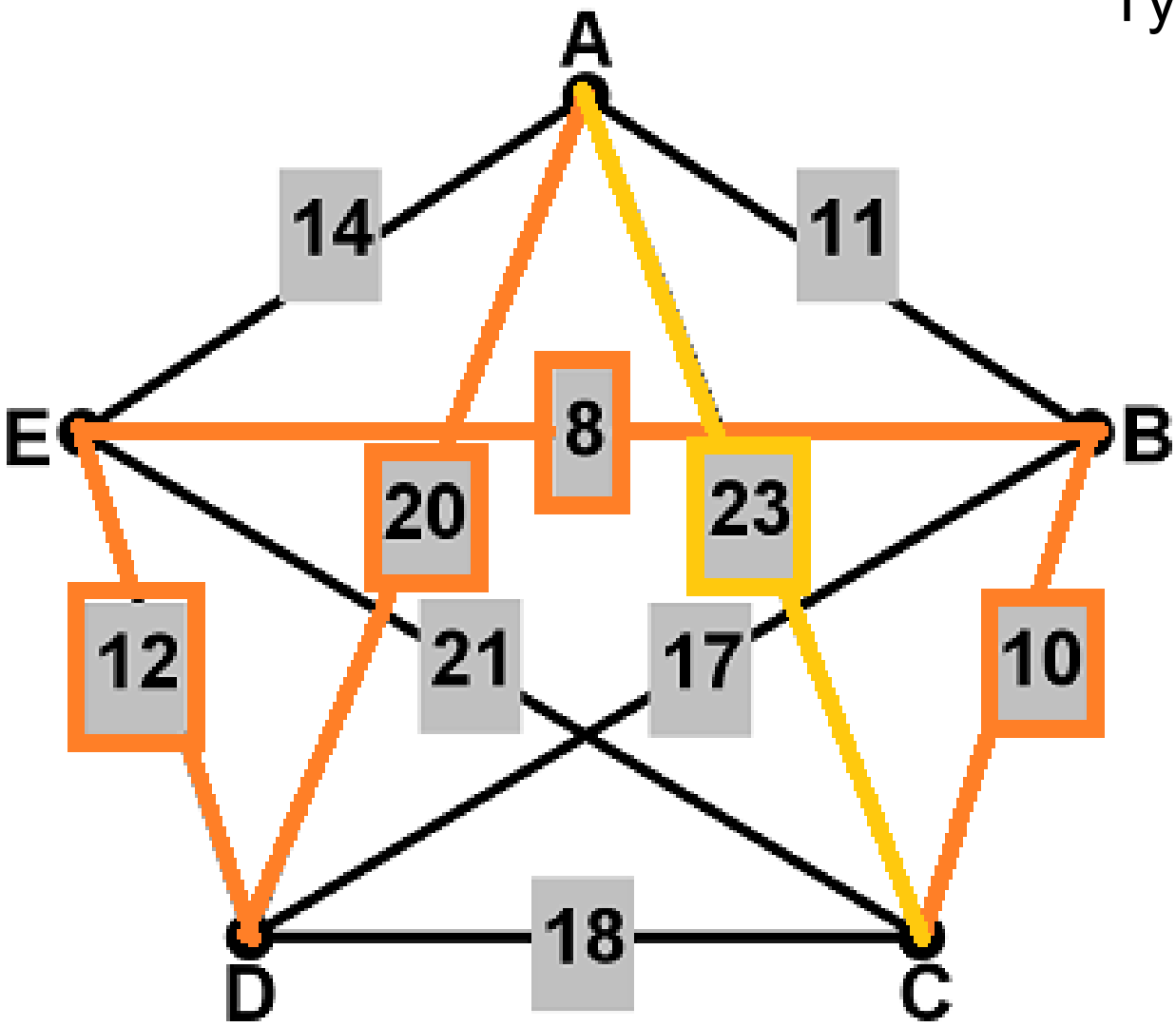


Edges Chosen: 8, 10, 12, 20, 23



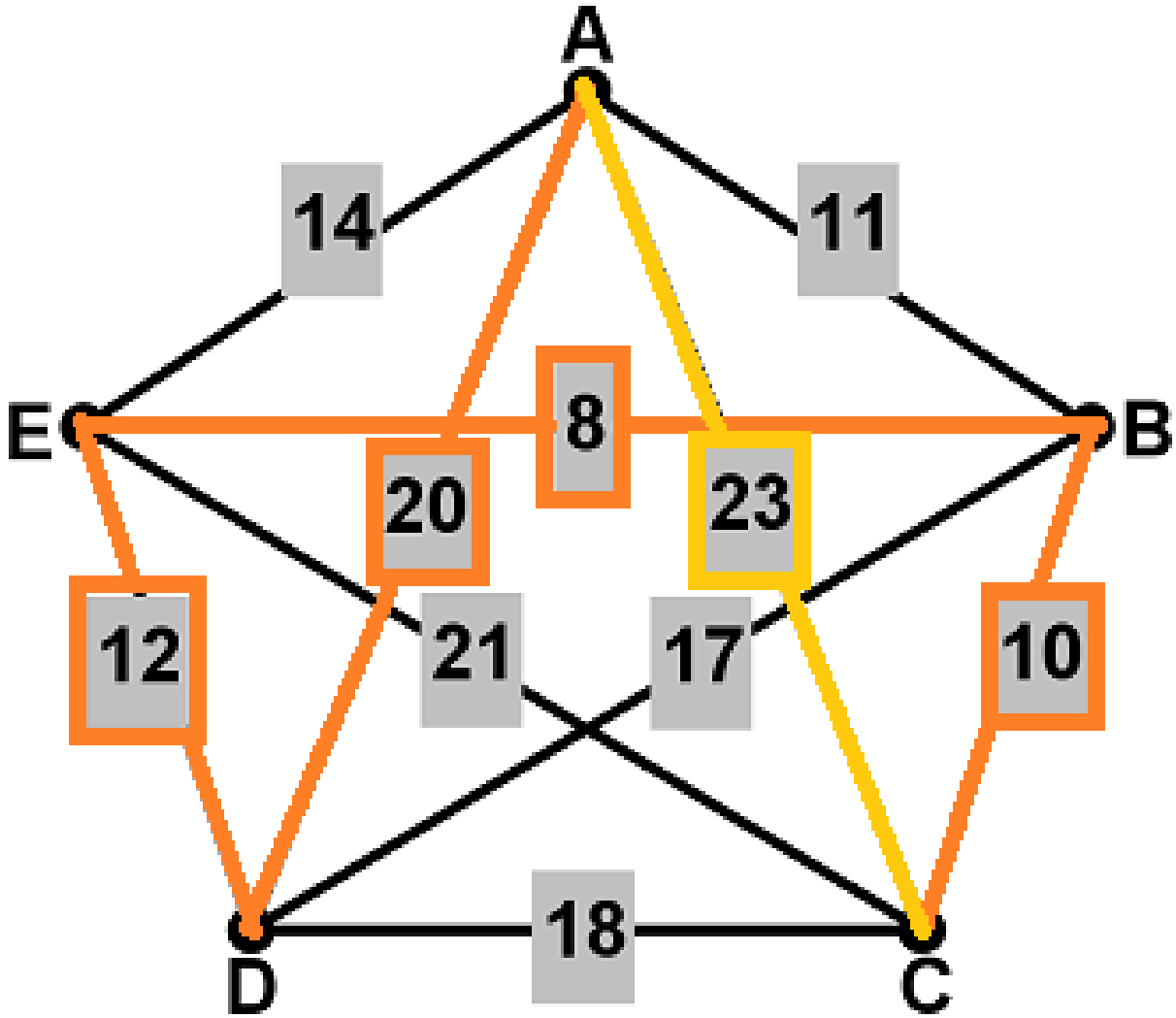
?(8.6) What is the Total Weight?

Type a number.



Edges Chosen: 8, 10, 12, 20, 23

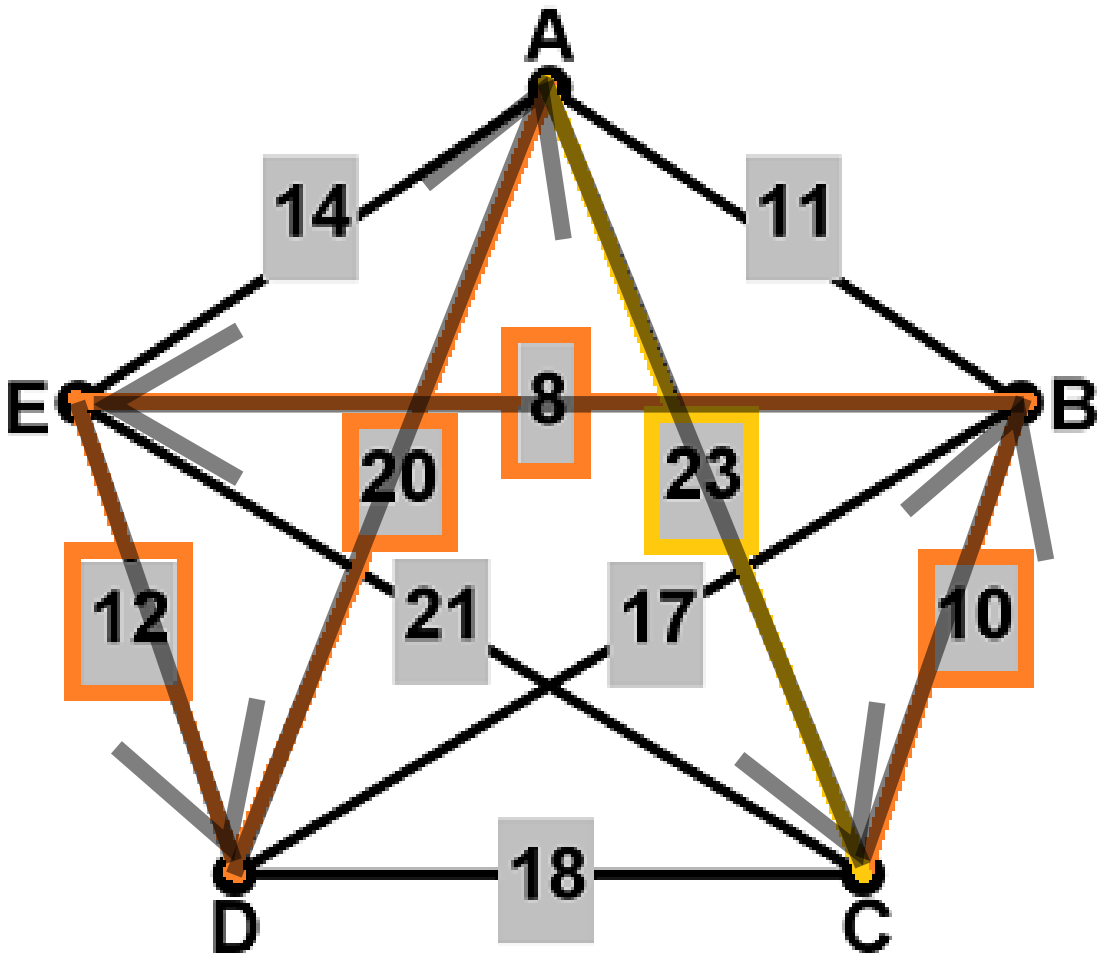
Total weight: $8 + 10 + 12 + 20 + 23 = 73$



Hamiltonian Circuit

We can start at any vertex, and follow the chosen edges to find our circuit.

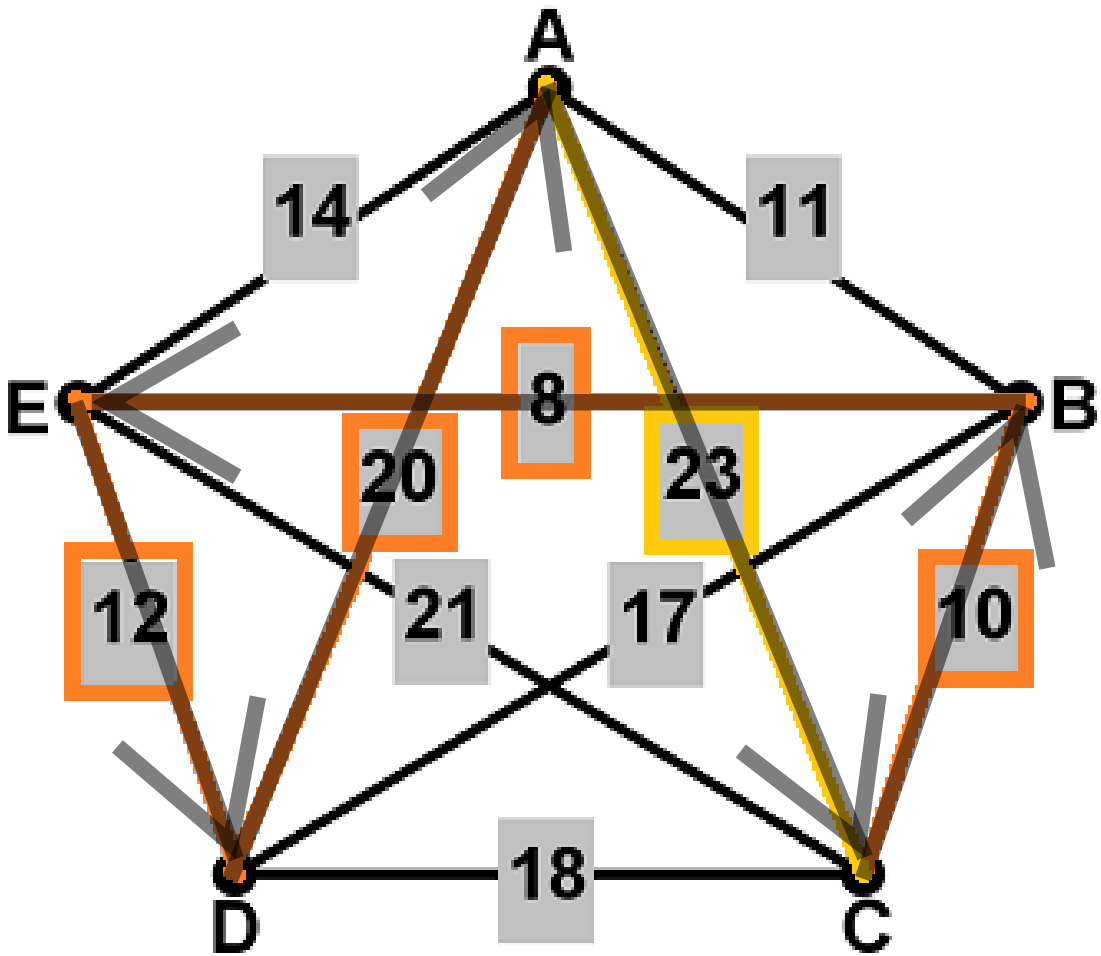
Example: A, C, B, E, D, A



Hamiltonian Circuit

We can start at any vertex, and follow the chosen edges to find our circuit.

Example: E, D, A, C, B, E



?(8.7) Shortest Hamiltonian Circuit

Is the circuit that Sorted Edges found the shortest Hamiltonian circuit on this graph?

Yes, or **No**?

Shortest Hamiltonian Circuit

Is the circuit that Sorted Edges found the shortest Hamiltonian circuit on this graph?

No!

- It has total weight 73
- The one we found on Monday, A, B, C, D, E, A, was shorter, with total weight 65.

Sorted Edges Algorithm Evaluation

The Sorted Edges Algorithm is:

- **Not Optimal:** It might not get the circuit with the lowest weight
- **Efficient:** It gives an answer quickly

The Sorted Edges Algorithm is a **heuristic algorithm**.
It is fast, and while its answer is not the best, it is still good.

Next Time

- Exam 4