Intro to Contemporary Math

Introduction to Graph Theory

Nicholas Nguyen
nicholas.nguyen@uky.edu

Department of Mathematics
UK

Announcements

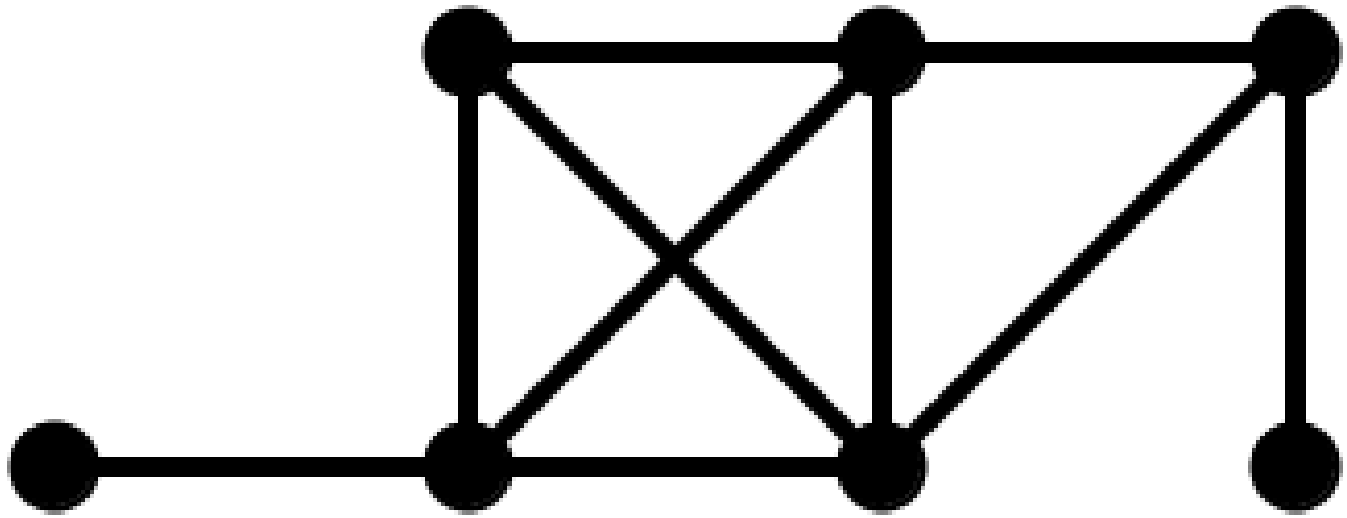
- Your project (all parts) must be uploaded on Canvas by November 20.
- There will be a homework assignment on WebWork (available Wednesday). It will be due November 26.

Introduction to Graph Theory

- A **graph** is a set of points called **vertices** (singular **vertex**) and lines called **edges** that connect some of the vertices.
- Application: Graphs can visually depict links between objects. The objects are drawn as vertices, and the links are represented with edges.

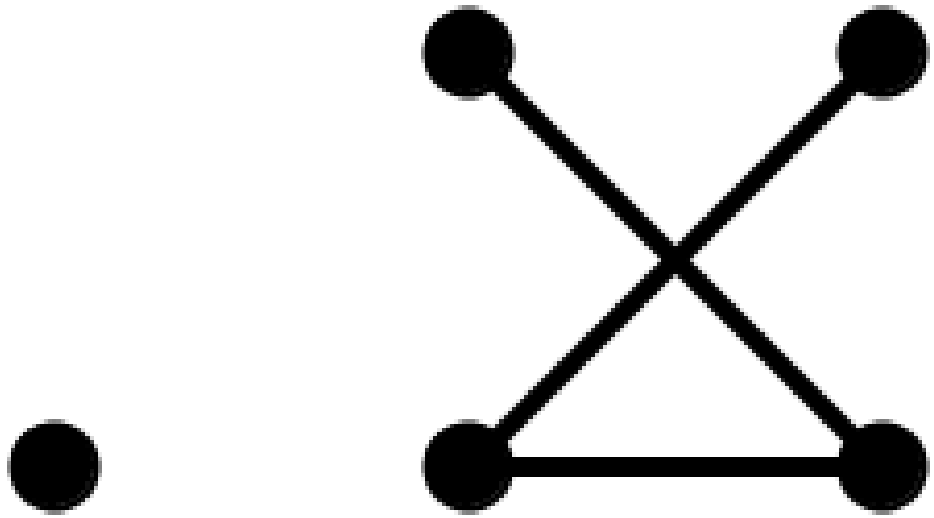
Graph Example 1

This is a graph with 7 vertices and 10 edges. **Edges can overlap.** There does not have to be a vertex at an overlap.



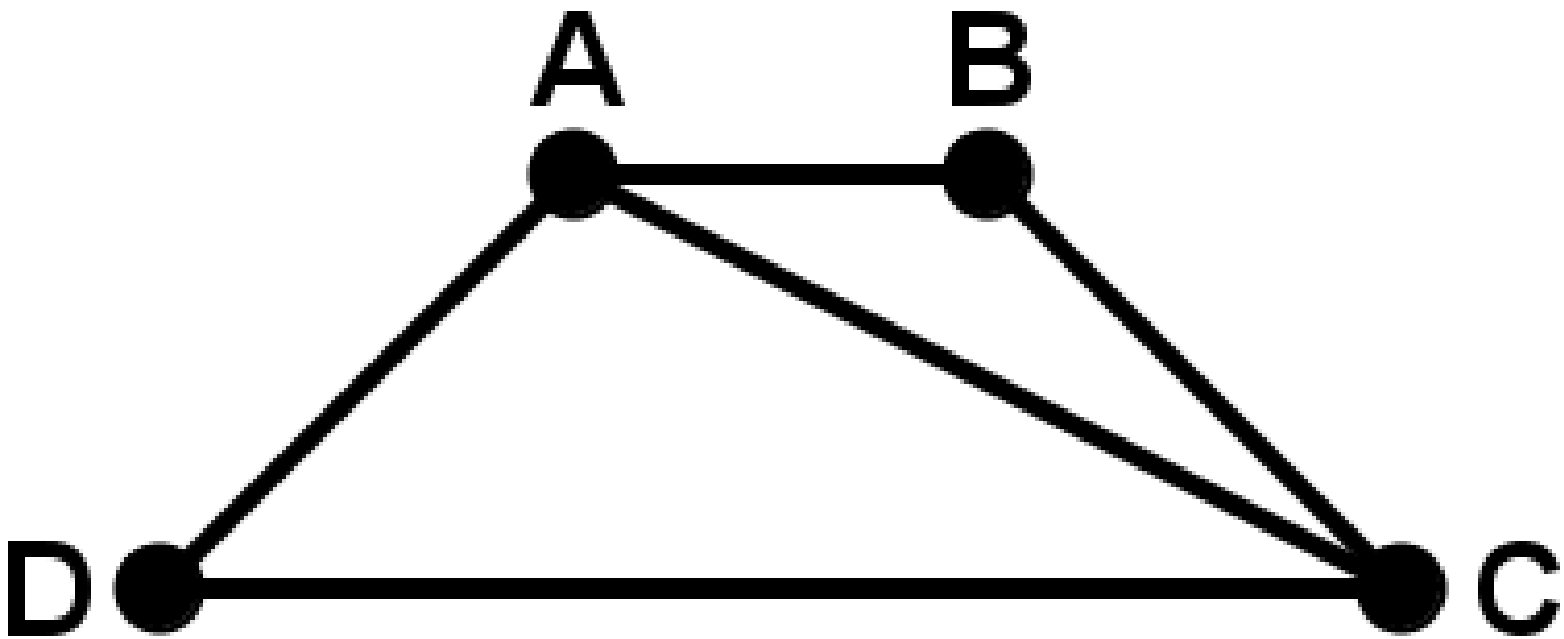
Graph Example 2

This is a graph with 5 vertices and 3 edges. Edges can overlap. Graphs can be in **multiple "pieces."** Vertices can be by themselves with no edges.



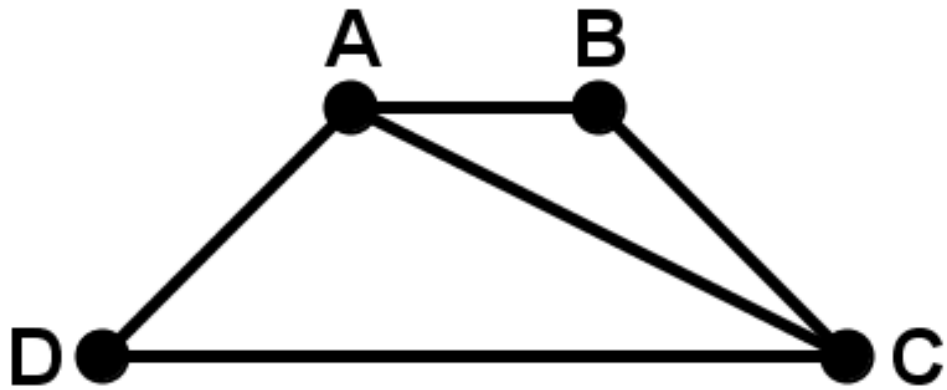
Describing Graphs: Labeling Vertices

This graph has 4 vertices and 5 edges.
We can label vertices to help distinguish them.



Describing Graphs: Vertex Sets

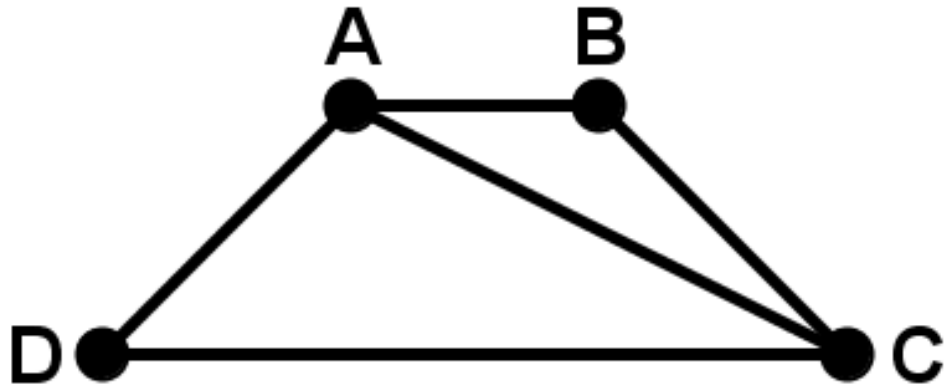
We can label vertices to help distinguish them.



Using these labels, we can write the **vertex set** of the graph: list the labels between braces:

$$\{A, B, C, D\}$$

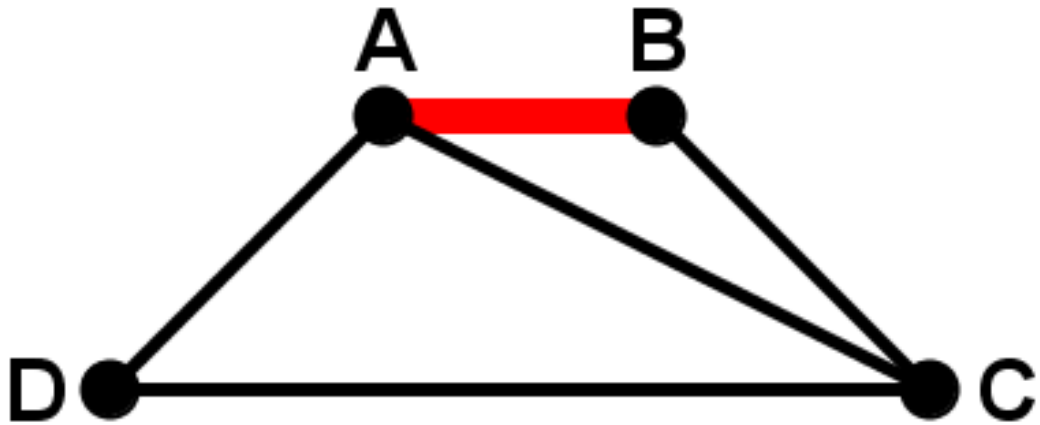
Describing Graphs: Edge Labels



Using the vertex labels, we can denote each edge using the vertices at its ends. The edge between A and B would be

$$\{A, B\}$$

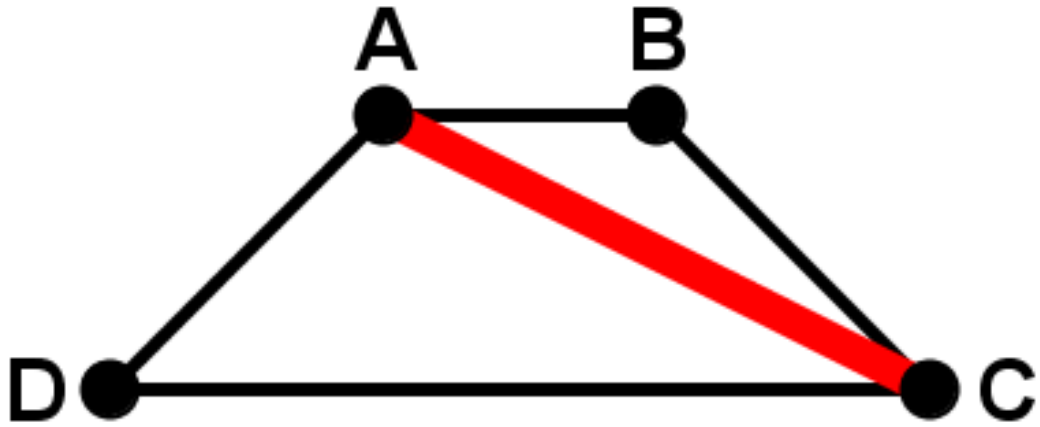
Describing Graphs: Edge Sets



The edge set of a graph is a list of all of its edges:

$$\{\{A, B\},$$

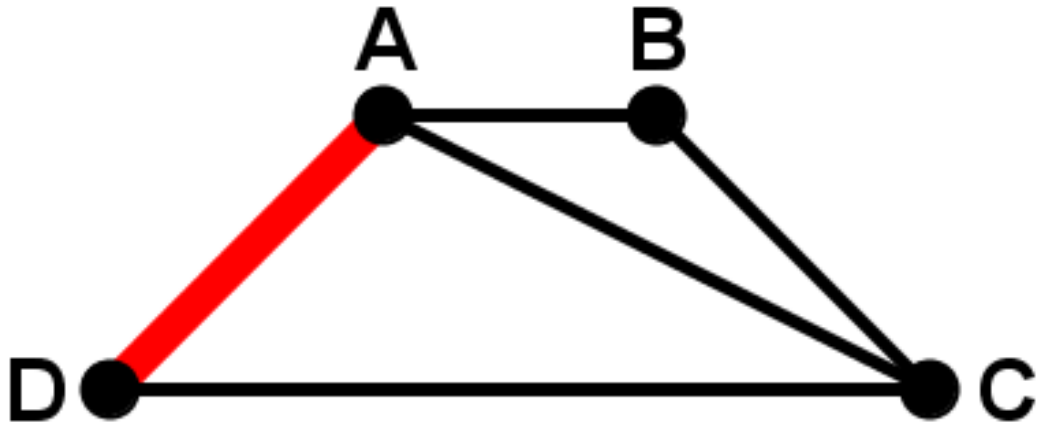
Describing Graphs: Edge Sets



The edge set of a graph is a list of all of its edges:

$$\{\{A, B\}, \{A, C\}, \quad \}$$

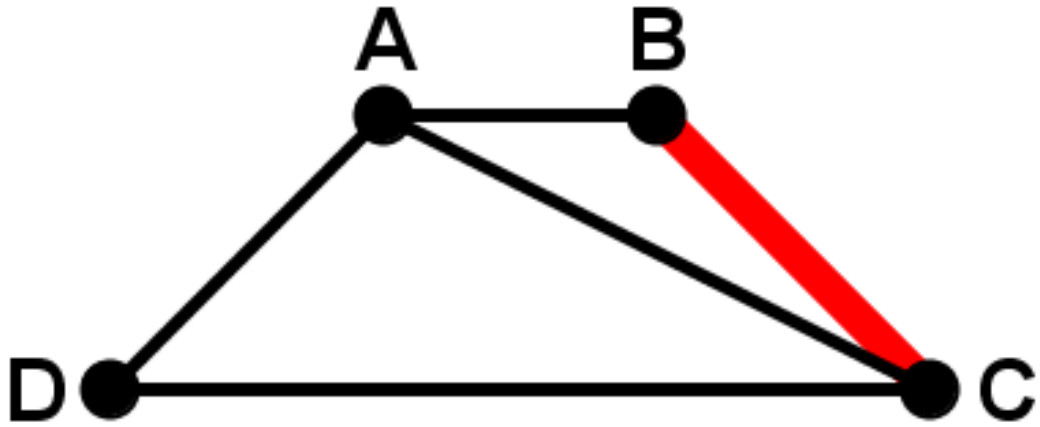
Describing Graphs: Edge Sets



The edge set of a graph is a list of all of its edges:

$$\{\{A,B\},\{A,C\},\{A,D\},\quad\}$$

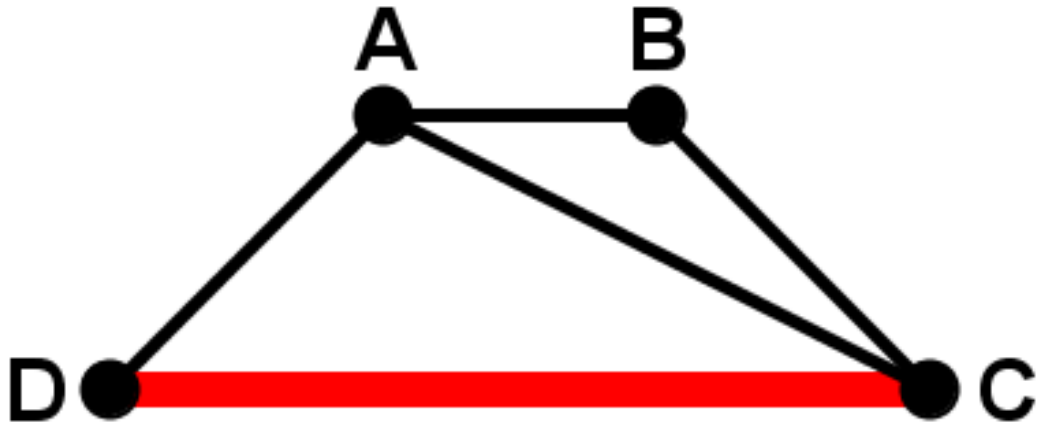
Describing Graphs: Edge Sets



The edge set of a graph is a list of all of its edges:

$$\{\{A,B\},\{A,C\},\{A,D\},\{B,C\},\quad\}$$

Describing Graphs: Edge Sets



The edge set of a graph is a list of all of its edges:

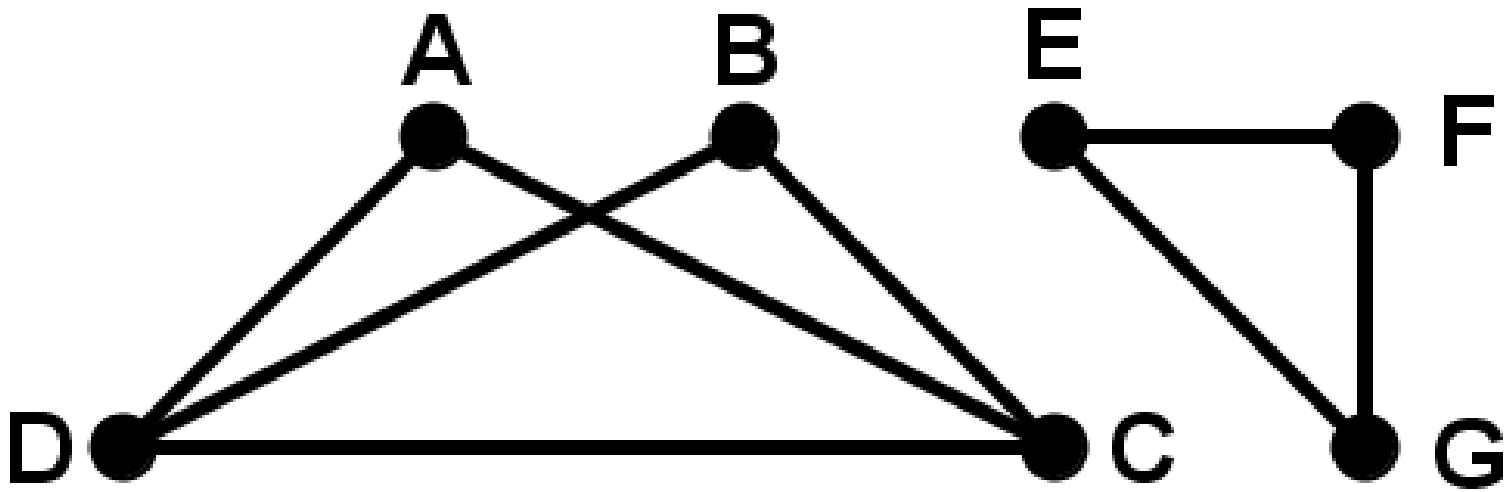
$$\boxed{\{\{A,B\},\{A,C\},\{A,D\},\{B,C\},\{C,D\}\}}$$

Graph Notation

- The letter v is the number of vertices.
- The **order** is another word for the number of vertices.
- The letter e is the number of edges.

?(1.1) Graph Notation, Vertex Sets, Edge Sets

For this graph, write the vertex set and edge set.



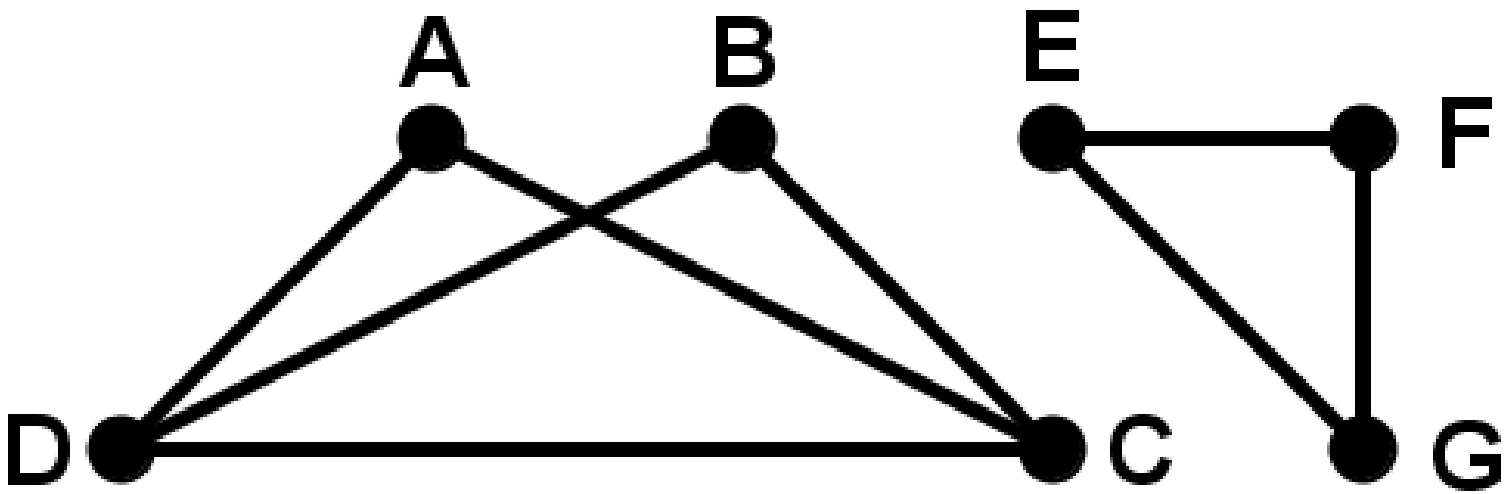
Then match:

1) v
2) e
3) order
4) Vertex set

A) 2
B) 4
C) 7
D) 8
E) {A, B, C, D, E, F, G}

Type and send four letters.

Graph Notation, Vertex Sets, Edge Sets

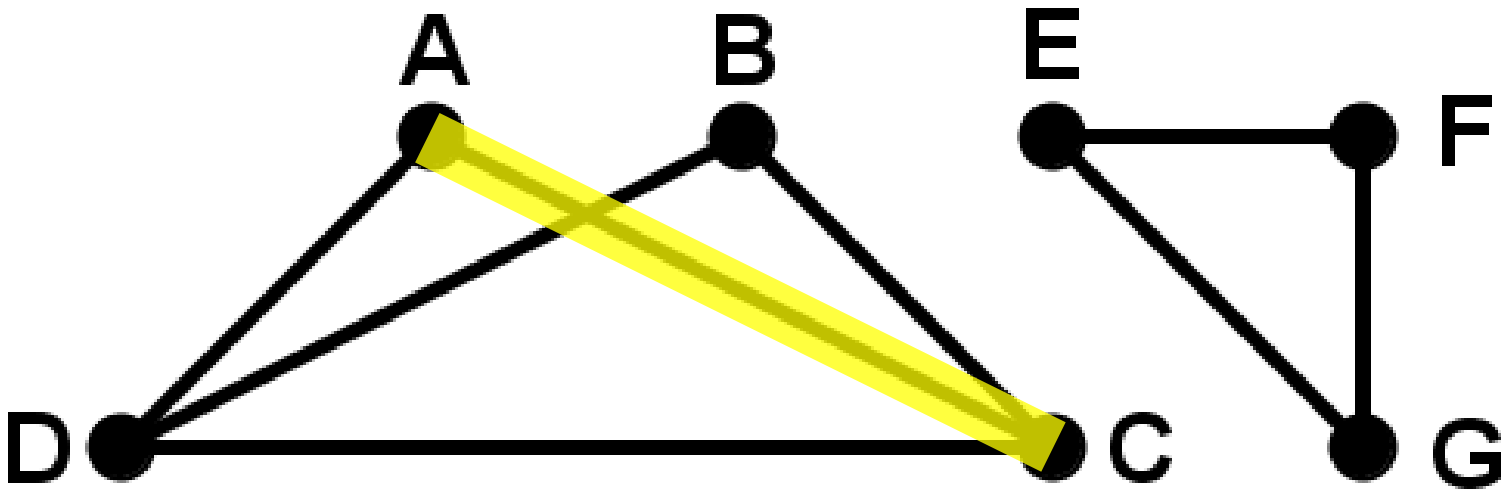


Vertex set:

$$\{A, B, C, D, E, F, G\}$$

7 vertices, so $v = 7$ (and the order equals 7)

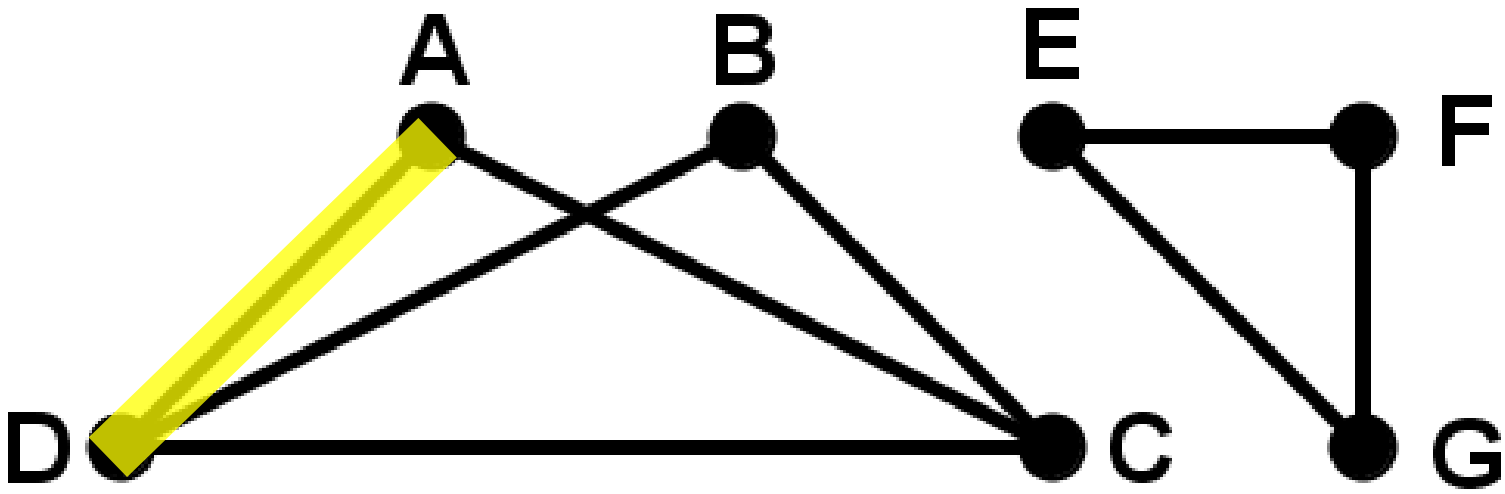
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$\{\{A, C\},$ $\}$

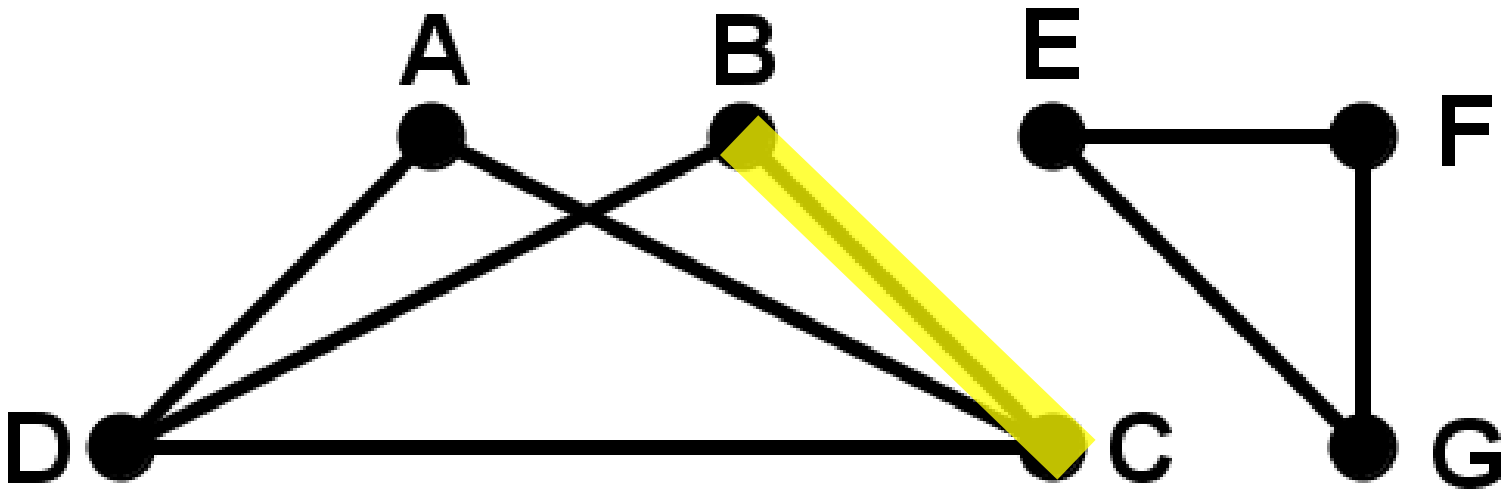
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$\{\{A, C\}, \{A, D\},$ $\}$

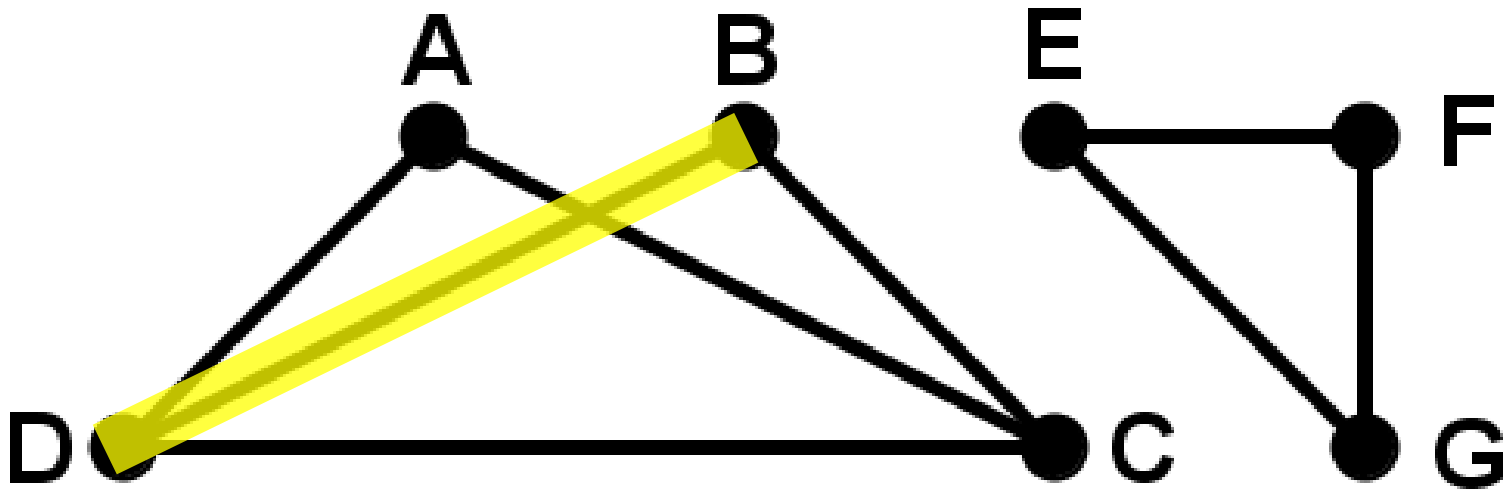
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$\{\{A, C\}, \{A, D\}, \{B, C\},$ $\}$

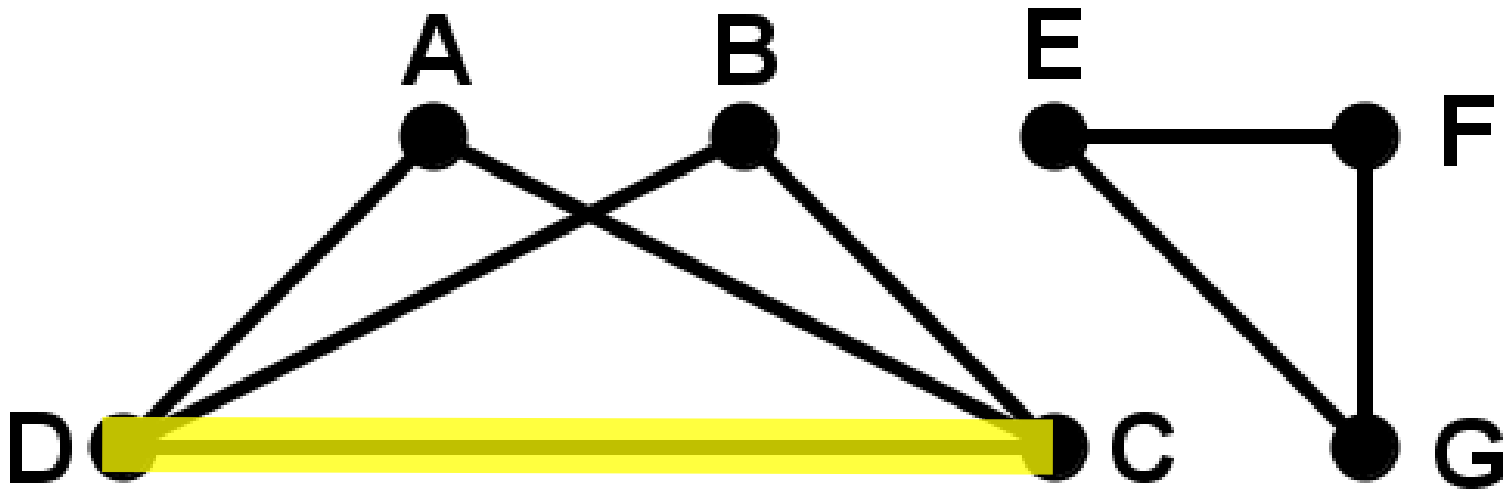
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\qquad\qquad\}$$

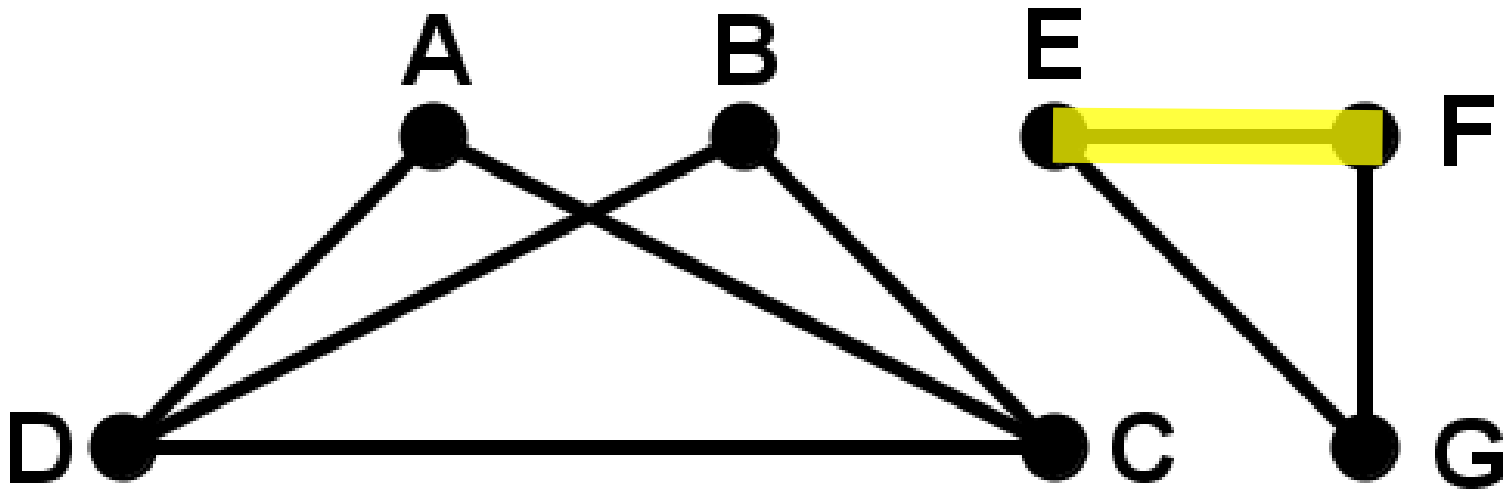
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\{C,D\},\qquad\qquad\}$$

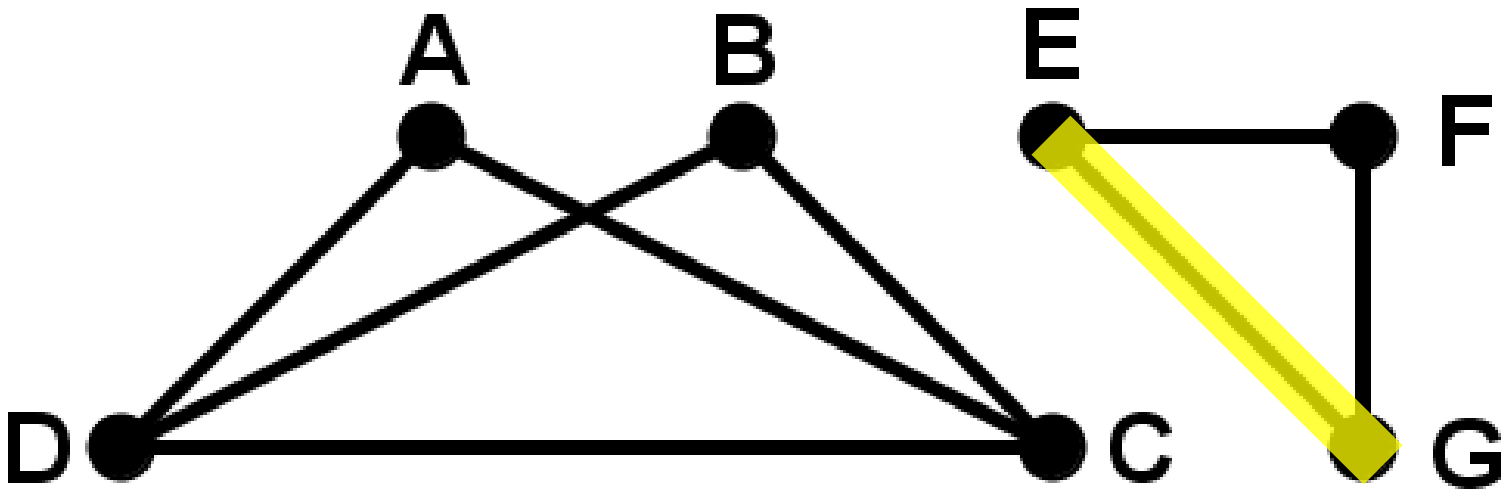
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\{C,D\},\{E,F\}\}$$

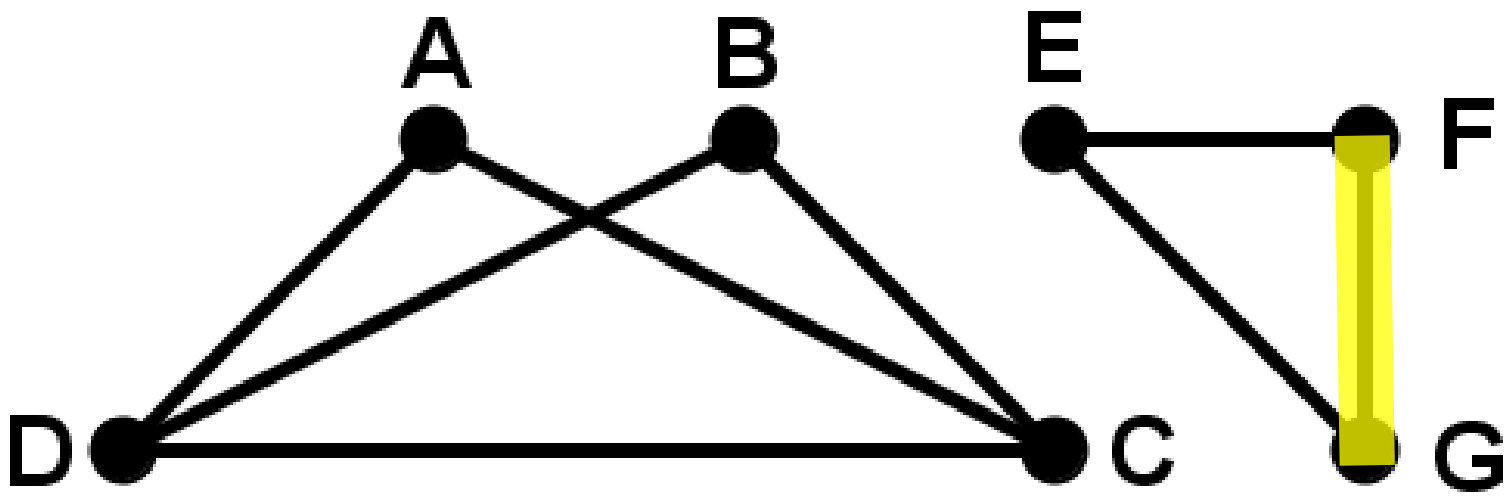
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\{C,D\},\{E,F\},\{E,G\} \quad \}$$

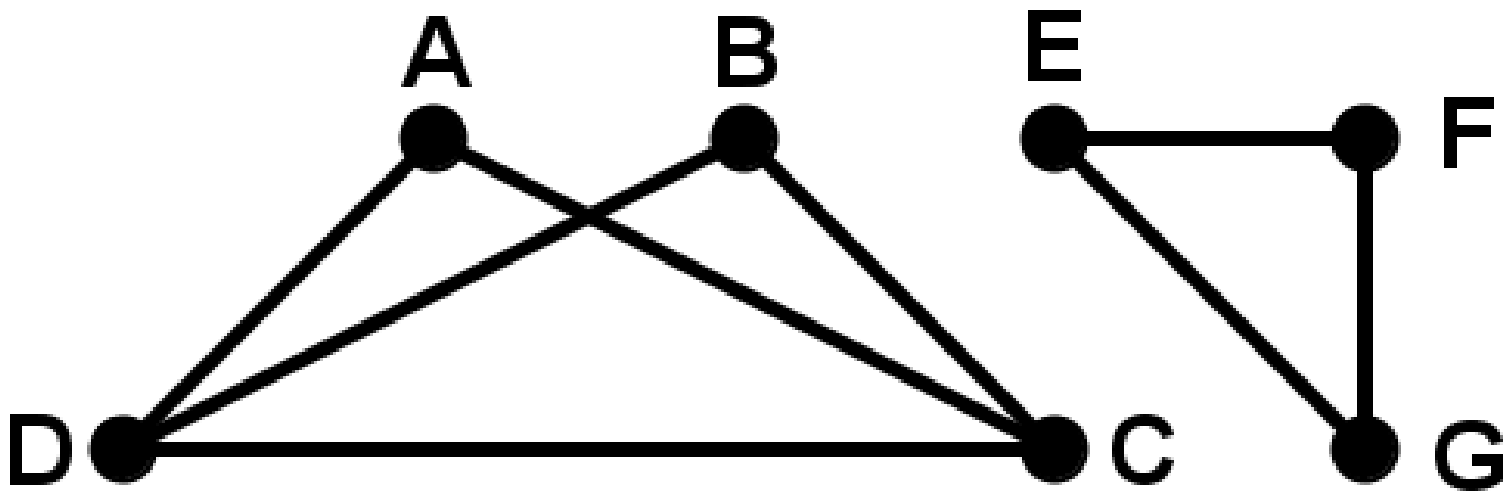
Graph Notation, Vertex Sets, Edge Sets



Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\{C,D\},\{E,F\},\{E,G\},\{F,G\}\}$$

Graph Notation, Vertex Sets, Edge Sets



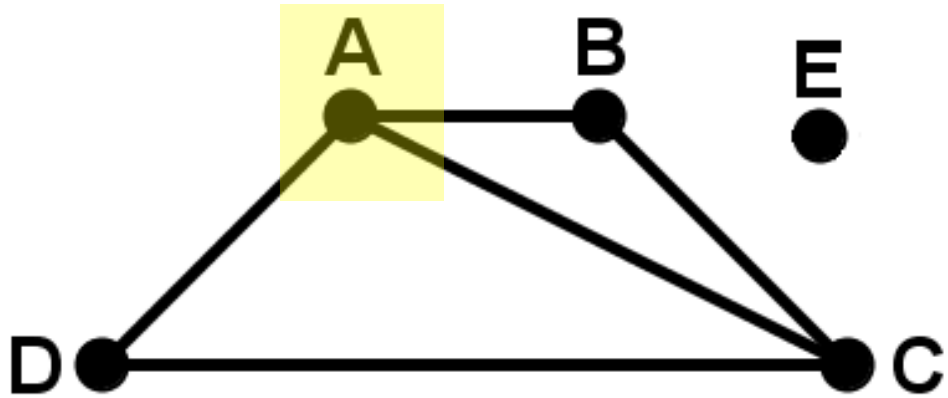
Edge set:

$$\{\{A,C\},\{A,D\},\{B,C\},\{B,D\},\{C,D\},\{E,F\},\{E,G\},\{F,G\}\}$$

8 edges, so $e = 8$

Degree of a Vertex

The **degree of a vertex** is the **number of edges attached to that vertex.**



- Vertex A has degree 3
-
-
-

Degree of a Vertex

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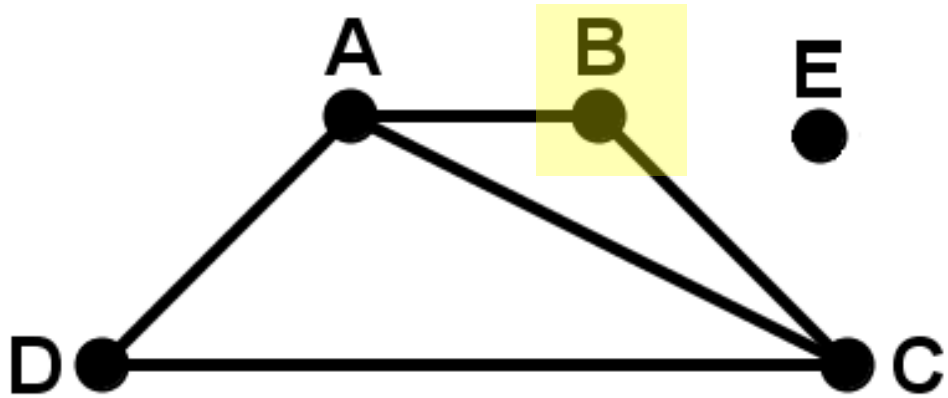

- Vertex A has degree 3
- Vertex B has degree 2
-
-
-

Degree of a Vertex

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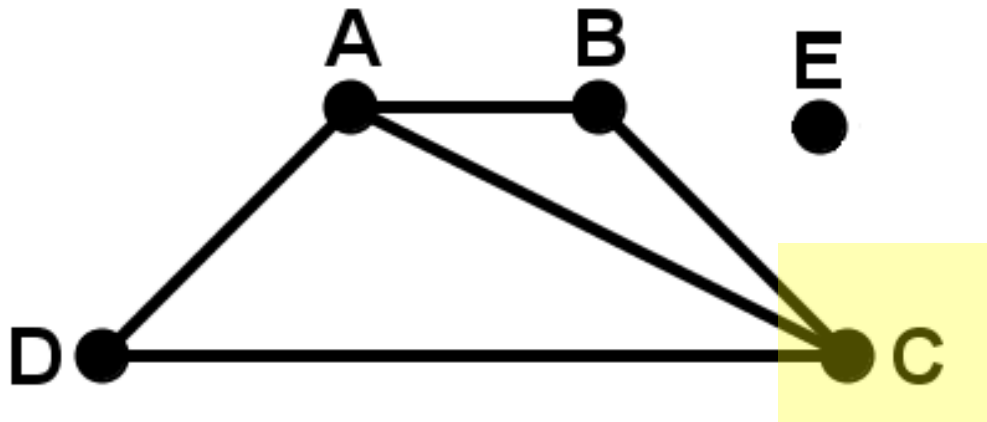
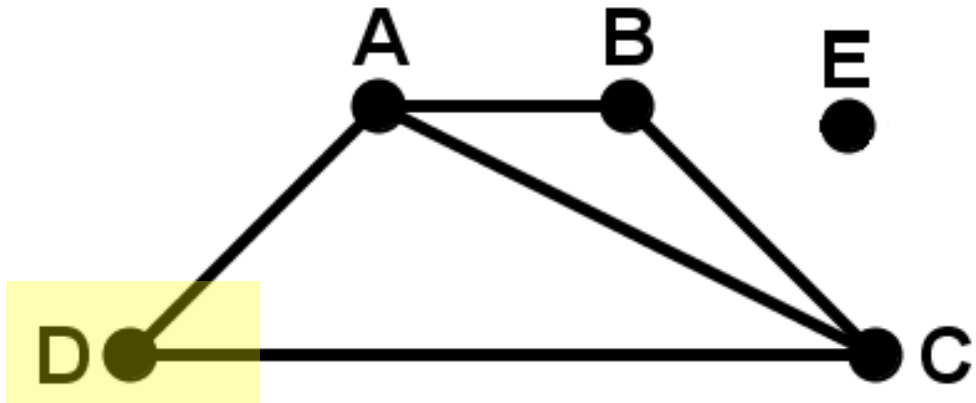


- Vertex A has degree 3
- Vertex B has degree 2
- Vertex C has degree 3

Degree of a Vertex

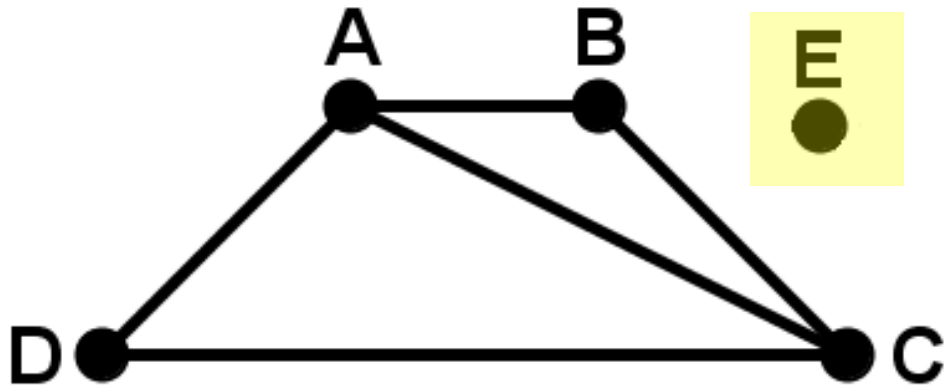
The **degree of a vertex** is the **number of edges attached to that vertex.**



- Vertex A has degree 3
- Vertex B has degree 2
- Vertex C has degree 3
- Vertex D has degree 2
-

Degree of a Vertex

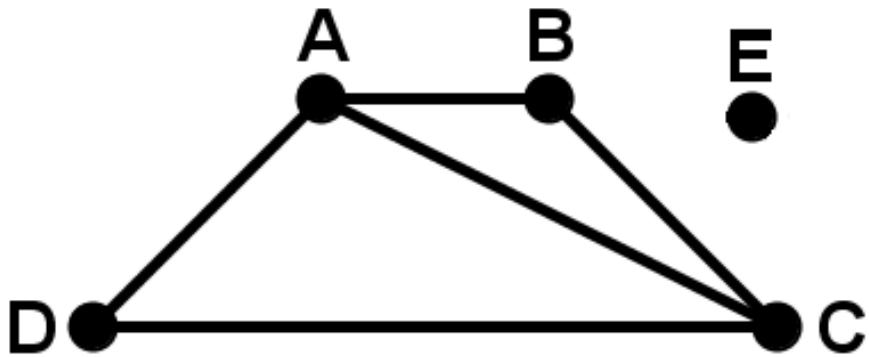
The **degree of a vertex** is the **number of edges attached to that vertex.**



- Vertex A has degree 3
- Vertex B has degree 2
- Vertex C has degree 3
- Vertex D has degree 2
- Vertex E has degree 0

Degree List

The **degree list** of a graph is a list of numbers which are the degrees of each vertex, ordered from smallest to largest.

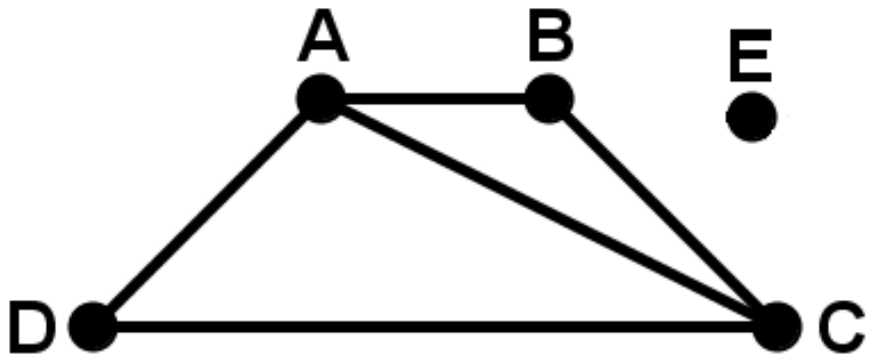


1. List the degrees of each vertex:

A	B	C	D	E
3	2	3	2	0

Degree List

The **degree list** of a graph is a list of numbers which are the degrees of each vertex, ordered from smallest to largest.



1. List the degrees of each vertex:

A	B	C	D	E
3	2	3	2	0

2. Sort degree numbers:

0,2,2,3,3

Next Time

- Isomorphic Graphs