Intro to Contemporary Math

Vertex Degrees and Isomorphic Graphs

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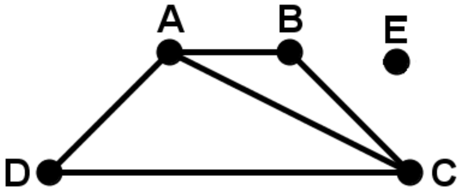
Degree of a Vertex

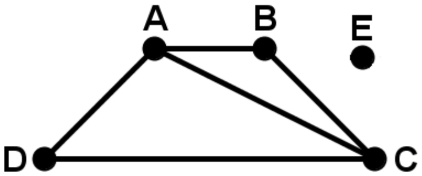
The **degree of a vertex** is the **number of edges attached to that vertex.**



- Vertex A has degree 3
- Vertex B has degree 2
- Vertex C has degree 3
- Vertex D has degree 2
- Vertex E has degree 0

Degree List

The **degree list** of a graph is a list of numbers which are the degrees of each vertex, ordered from smallest to largest.



1. List the degrees of each vertex:

A	B	C	D	E
3	2	3	2	0

2. Sort degree numbers:

$\boxed{0,2,2,3,3}$

Announcements

- Your project (all parts) must be uploaded on Canvas by November 20.

- Homework posted; due November 26.
- Mini-Exam 4 is on November 28.

Isomorphic Graphs

Two graphs are **isomorphic** if the vertices are connected the same way in the graphs.

- We can relabel/move the vertices of one graph and place them on top of the second to make the edges line up.
-

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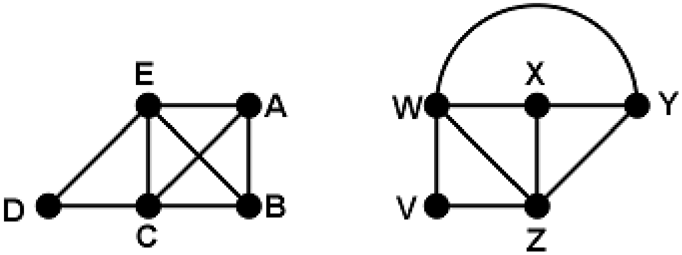

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- We can relabel/move the vertices of one graph and place them on top of the second to make the edges line up.
- **IF** two graphs are isomorphic, then a **graph isomorphism** tells us how to move and relabel the vertices of the first graph to make it look like the second graph.

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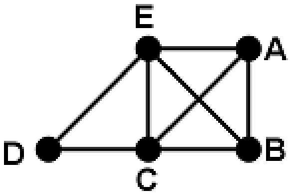

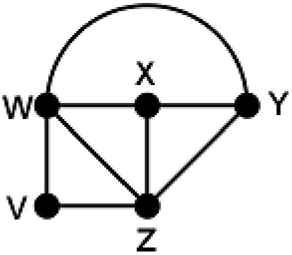

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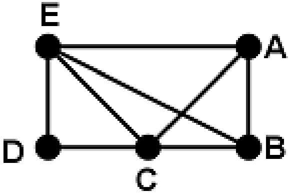

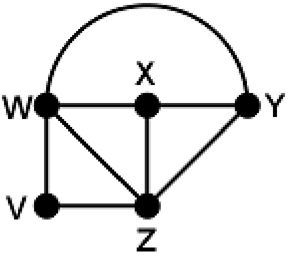

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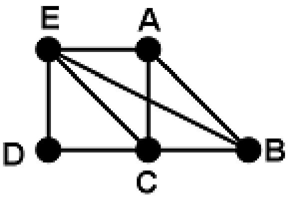

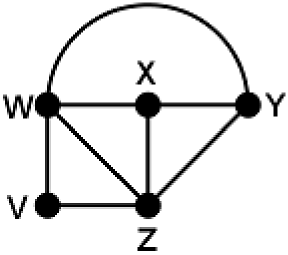

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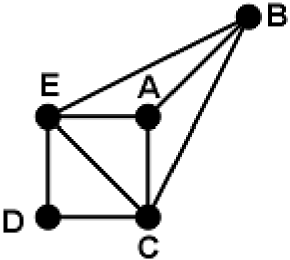

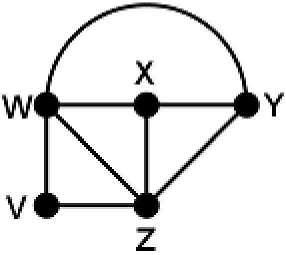

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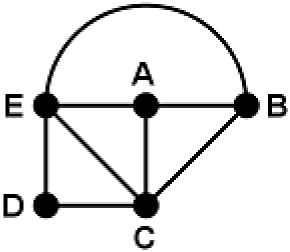

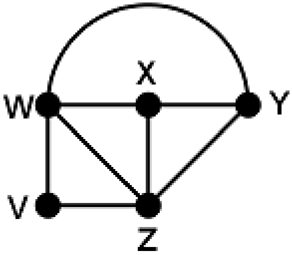

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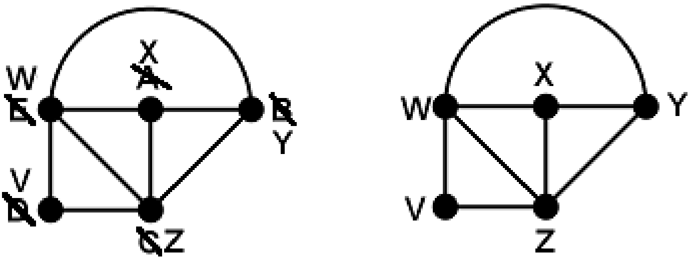


Graph isomorphism: A to X, B to Y, C to Z, D to V, E to W

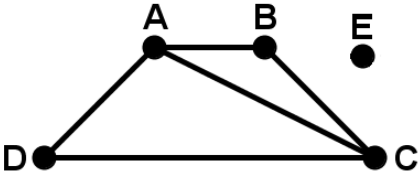
Quick Check: Not Isomorphic

If the graphs have a different number of vertices or edges, or have different degree lists, they are not isomorphic.

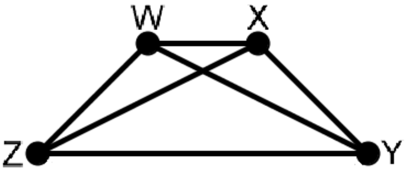
Number of Edges Must Be Equal

These graphs are not isomorphic.
Vertex and edge counts are different.

$v = 5, \; e = 5$

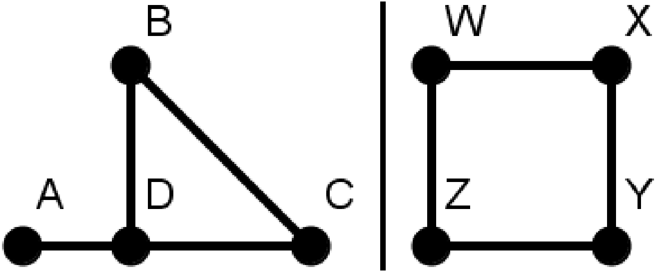


$v = 4, \; e = 6$



Degree Lists Must Match

These graphs are not isomorphic:



Degree lists are different:

Left: 1, 2, 2, 3 (A, B, C, D)
Right: 2, 2, 2, 2 (W, X, Y, Z)

Relabeling Vertices

If the two graphs are isomorphic, you can relabel (move) a vertex in the 1st graph to a vertex in the 2nd graph. You must obey these rules:

1.
2.

Relabeling Vertices

If the two graphs are isomorphic, you can relabel (move) a vertex in the 1st graph to a vertex in the 2nd graph. You must obey these rules:

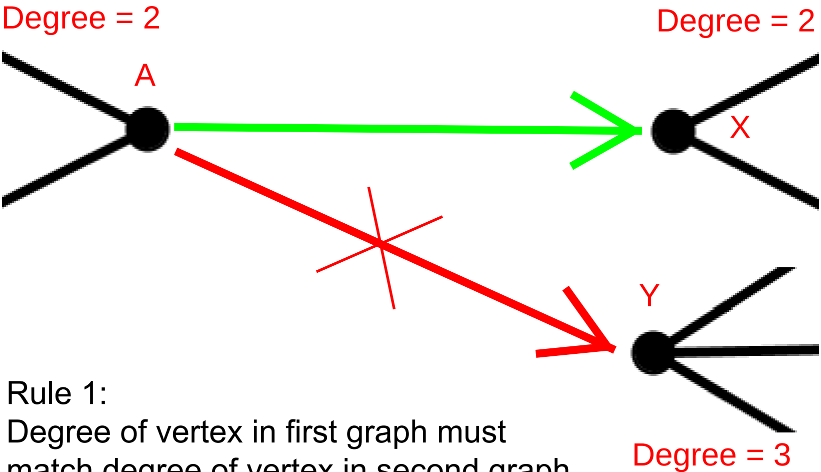
1. Degree of vertex in 1st graph must match degree of vertex in 2nd graph
2.

Relabeling Vertices

If the two graphs are isomorphic, you can relabel (move) a vertex in the 1st graph to a vertex in the 2nd graph. You must obey these rules:

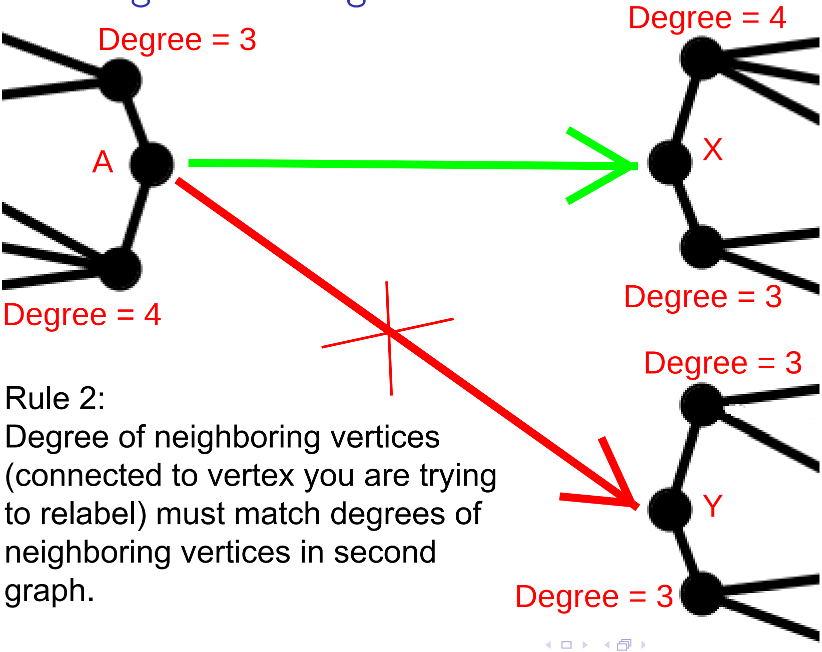
1. Degree of vertex in 1st graph must match degree of vertex in 2nd graph
2. Degrees of neighboring vertices (connected to the vertex you are trying to relabel) must match degrees of neighboring vertices of vertex in 2nd graph

Match Degrees



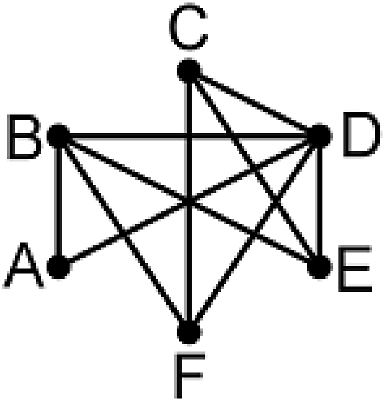
Rule 1:
Degree of vertex in first graph must match degree of vertex in second graph.

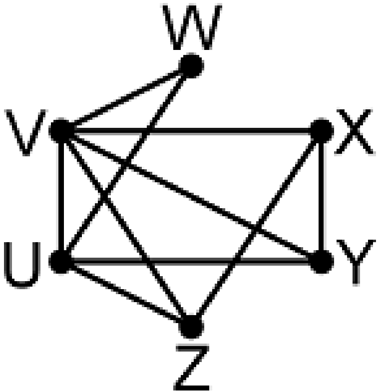
Match Degrees of Neighbors



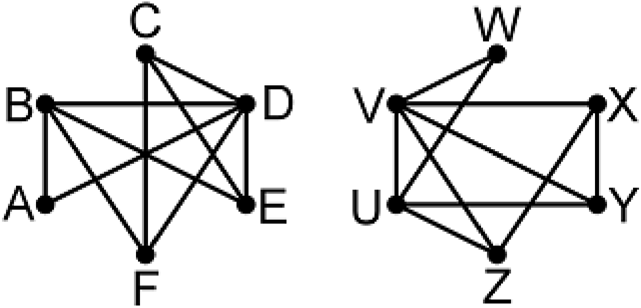
Rule 2:
Degree of neighboring vertices (connected to vertex you are trying to relabel) must match degrees of neighboring vertices in second graph.

Isomorphic Graphs?



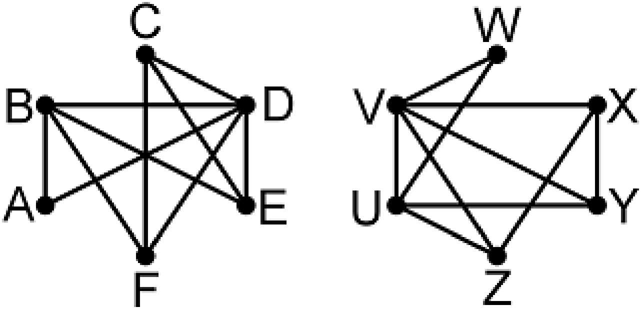


?(2.1) Relabeling Example



- Find the degree lists of both graphs.
- Which vertex in the 2nd graph should **A** be relabeled to?
  Type and send a letter U-Z.
  Notice that A has degree 2.

Relabeling Example Degree Lists



- Degree list:

  $$2, 3, 3, 3, 4, 5$$

  $$A, C, E, F, B, D$$

- Degree list:

  $$2, 3, 3, 3, 4, 5$$

  $$W, X, Y, Z, U, V$$

Relabeling Example (Vertex A)

- Degree list:

  $2, 3, 3, 3, 4, 5$

  A, C, E, F, B, D

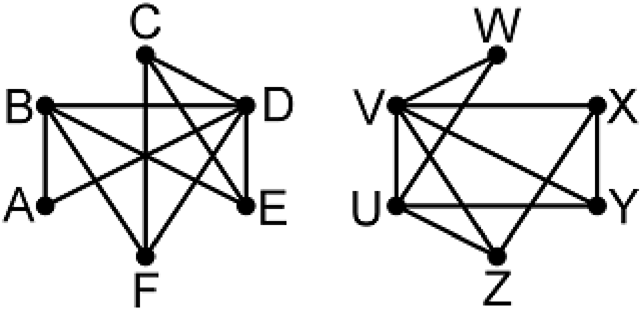
- Degree list:

  $2, 3, 3, 3, 4, 5$

  W, X, Y, Z, U, V

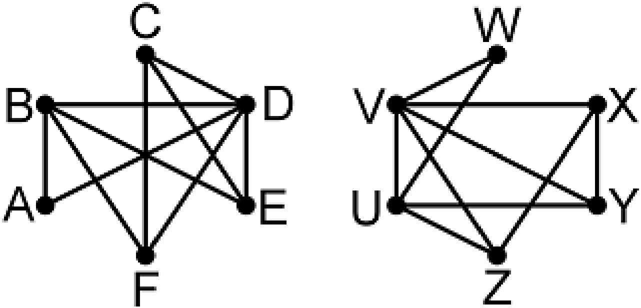
If a number appears **only once** in each graph's degree list, the vertex in the 1st graph with that degree will be **relabeled** to the vertex in the 2nd graph with that degree. Hence, we should relabel vertex **A** to vertex **W**.
(We also must relabel **B** to **U** and **D** to **V**).

?(2.2) Relabeling Example



- What are the degrees of the vertices connected to **C**?
- Which vertex in the 2nd graph should **C** be relabeled to? Type and send a letter U-Z.
- There's only one correct answer. Check the neighbors.

Relabeling Example Degrees of Neighbors of C

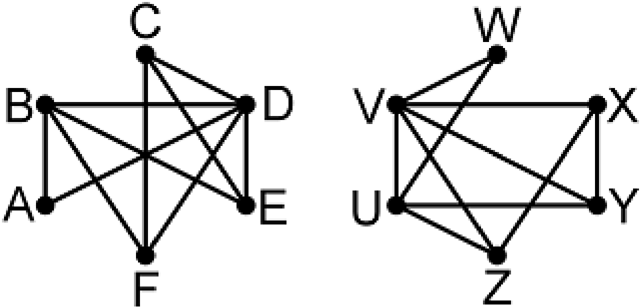


C connects to vertices of degree 3, 3, 5 (E, F, D)

- Which vertex in the 2nd graph should **C** be relabeled to?

X, Y, and Z all have degree 3 like C does.
Let us look at the neighbors of X, Y, and Z.

Relabeling Example (Vertex C)



C connects to vertices of degree 3, 3, 5 (E, F, D)

The right graph has three vertices with degree 3: X, Y, and Z.

- X connects to vertices of degree 3, 3, 5 (Y, Z, V)
- Y connects to vertices of degree 3, 4, 5 (X, U, V)
- Z connects to vertices of degree 3, 4, 5 (X, U, V)

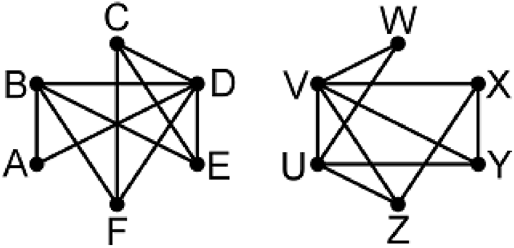
Relabeling Example (Vertex C)

C connects to vertices of degree 3, 3, 5 (E, F, D)
The right graph has three vertices with degree 3: X, Y, and Z.

- X connects to vertices of degree 3, 3, 5 (Y, Z, V)
- Y connects to vertices of degree 3, 4, 5 (X, U, V)
- Z connects to vertices of degree 3, 4, 5 (X, U, V)

Vertex **C** must be relabeled to **X**. Both connect to vertices of degrees 3, 3, 5.

Bonus: The Entire Isomorphism



A	to	W
B	to	U
C	to	X
D	to	V
E	to	Y
F	to	Z

Next Time

- Planar Graphs: No Overlaps