Intro to Contemporary Mathematics

Extra Practice

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? (3.7) Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. How many **faces** does it have?

A) 52
B) 26
C) 13
D) 8
E) 5
F) 4

Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. How many **faces** does it have?

The list of face degrees has 5 entries, so there must be

5 faces.

? (3.8) Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. How many **edges** does it have?

A) 52
B) 26
C) 13
D) 8
E) 5
F) 4

Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. How many **edges** does it have?
Add up the degrees of the faces and divide by two:

$$3 + 4 + 5 + 6 + 8 = 26$$

$$26 \div 2 = \boxed{13 \text{ edges}}$$

? (3.9) Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. We know it has 5 faces and 13 edges. How many vertices does it have?

A) 8
B) 10
C) 20
D) -16
E) -6
F) 5

Degree List

A graph has faces with degrees 3, 4, 5, 6, 8. We know it has 5 faces and 13 edges. How many vertices does it have?

$$e = 13,\ f = 5$$

$$v - e + f = 2$$

$$v - 13 + 5 = 2$$

$$v = 2 + 13 - 5 = \boxed{10 \text{ vertices}}$$

? (3.10) Possible Degree List

Which of these lists of numbers can be a degree list (for the vertices) of a graph with 7 vertices?

A) 2, 3, 4, 4, 7, 7, 8
B) 2, 3, 4, 4, 7, 7, 9
C) 1, 3, 4, 4, 7, 7, 9
D) 2, 3, 4, 4, 6, 6, 6
E) 2, 2, 3, 4, 4, 4, 5, 6
F) 2, 3, 4, 4, 4, 5, 6

Hint: In each list, add up the degrees and divide by 2. Can you get a fraction for the number of edges?

Hint: Can a vertex have a degree higher than the number of other vertices in the graph? Can a vertex connect to more vertices than there are in the graph?

Possible Degree List

Which of these lists of numbers can be a degree list (for the vertices) of a graph with 7 vertices?
F) 2, 3, 4, 4, 4, 5, 6
This is the only choice where:

- There are 7 entries, so there are 7 vertices.
- If we add up the degrees and divide by two, we get a whole number:

$$\frac{2+3+4+4+4+5+6}{2} = \frac{28}{2} = 14 \text{ edges}$$

- Each vertex's degree is less than or equal to 6. In a graph with 7 vertices, a vertex can only have up to 6 edges connecting it to some (or all) of the other 6 vertices.

Possible Degree List

Which of these lists of numbers can be a degree list (for the vertices) of a graph with 7 vertices?

A) 2, 3, 4, 4, 7, 7, 8

B) 2, 3, 4, 4, 7, 7, 9

C) 1, 3, 4, 4, 7, 7, 9

These cannot be lists of degrees of vertices in a graph with 7 vertices. In a graph with 7 vertices, a vertex can only have up to 6 edges connecting it to some (or all) of the other 6 vertices, so each vertex's degree has to be less than or equal to 6.

Possible Degree List

Which of these lists of numbers can be a degree list (for the vertices) of a graph with 7 vertices?

D) 2, 3, 4, 4, 6, 6, 6

If we add up the degrees and divide by two, we do **not** get a whole number:

$$\frac{2+3+4+4+6+6+6}{2} = \frac{31}{2} = 15.5 \text{ edges?}$$

Graphs cannot have half of an edge!

Possible Degree List

Which of these lists of numbers can be a degree list (for the vertices) of a graph with 7 vertices?

E) 2, 2, 3, 4, 4, 4, 5, 6

This list has 8 entries. It must be the degree list of a graph with 8 vertices, not 7 vertices.

Next Time

- We will study graphs with extra information on the edges called weights.