Intro to Contemporary Mathematics

Euler's Formula Examples

Dr. Nguyen
nicholas.nguyen@uky.edu

Department of Mathematics
UK

Euler's Formula

For a connected planar graph with v vertices, e edges, and f faces, we have

$$v - e + f = 2.$$

This formula can be used if you are given two out of three measurements for the number of vertices, edges, or faces. Plug the known measurements into the formula and solve for the third missing measurement.

Example 1: Find v

A graph has 25 faces and 40 edges. How many vertices does it have?

Example 1: Find v

A graph has 25 faces and 40 edges. How many vertices does it have?

$$e = 40, \ f = 25$$

$$\begin{aligned} v - e + f &= 2 \\ v - 40 + 25 &= 2 \\ v - 15 &= 2 \\ v &= 2 + 15 = \boxed{17 \ \textit{vertices}} \end{aligned}$$

Example 2: Find e

A graph has 20 vertices and 7 faces. How many edges does it have?

Example 2: Find e

A graph has 20 vertices and 7 faces. How many edges does it have?

$$v = 20,\ f = 7$$

$$\begin{aligned} v - e + f &= 2 \\ 20 - e + 7 &= 2 \\ -e + 27 &= 2 \\ -e &= 2 - 27 \\ -e &= -25 \\ e &= \boxed{25 \text{ edges}} \end{aligned}$$

Example 3: Find f

A graph has 32 vertices and 45 edges. How many faces does it have?

Example 3: Find f

A graph has 32 vertices and 45 edges. How many faces does it have?

$$v = 32, \ e = 45$$

$$\begin{aligned} v - e + f &= 2 \\ 32 - 45 + f &= 2 \\ -13 + f &= 2 \\ f &= 2 + 13 = \boxed{15 \ \textit{faces}} \end{aligned}$$